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Effect of the Training Program on Nurses Knowledge and Practice Regarding Neonatal Sepsis in Intensive Care Unit, in Military and Police Hospitals, Khartoum State Sudan. Rabab.Mohamed Adam, Buthina Bassyonie Elssayed

Determination of Elements in Soil and Croups (Wheat and Sun flower) Using X-MET 5000 . Fawaz Taha Omer1, Alfadil Mahmoud Yousif2 & Fawzi Mohamed Ahmed3

Basic Pharmacology of Losartan Review Article. Muhanad S. Saad, Abdelwhab Hassan Mohamed, Mazin S. Abdalla Mohamed , Fath Elrahman A. Idris

The effect of Black Cumin (Nigella sativa L.) and Garlic Allium sativum L.) Oils on Dermatophytosis. Tayseer H.A. Mohammed, Khalid E.M. Idris, Elebaid M. Y. Elhaj

The effectiveness of the Random Forest algorithm in monitoring abnormal withdrawals to detect credit cards frauds. Hamza Mohamed Hassan¹, Awad Haj Ali Ahmed²

On Proof of Sharp Sobolev Embeddings. Isam Eldin Ishag Idris

رقم الإيداع

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Effect of the Training Program on Nurses Knowledge and Practice Regarding Neonatal Sepsis in Intensive Care Unit, in Military and Police Hospitals, Khartoum State Sudan

Rabab.Mohamed Adam¹, Buthina Bassyoni Elssayed²

¹Kamllin collage, Sudan

²Faculty of Nursing, Al-butana University, Sudan

Corresponding email: rababi145@gmail.com

Abstract

Neonatal sepsis is a significant cause of morbidity and mortality and it's considered to be one of the important unsolved problems around the world. Lack of knowledge regarding Neonatal sepsis control among nurses decreases compliance with these practices around the world. The aim of the study is to Know the effect of the Training Program on nurses' knowledge and Practice regarding care of neonate with sepsis in Obstetrics and Gynecology hospitals, Khartoum State, Sudan. A quasi experimental study was used. The study was conducted in military and police Hospitals in Khartoum state. 47 Nurses were included in the study after fulfilling the criteria of selection and agreed to participate in the study .The necessary research data were collected from them using a structured interview and a checklist pre and three months post intervention .Training program was offered to the Nurses about neonatal sepsis it was includ (definition ,causes, treatment, ,nursing care, incubator care) Data were analyzed by SPSS version 20 descriptive, and inferential statistic(T.test and chi squared) was used with significant P. value <0.05. Participants were all of them female and the most common age group between 20 -30 years, was 35(74.5%) with mean 27.7 ± 5.3 . Majority of them BSc nursing holder 40 (85.1%).Their experiences varies ,but the most common were between ,1-5 years and 6-10 years 15(32%) and 14(29.8%) respectively. Nurses overall Knowledge level

1

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regarding neonatal sepsis was 94% in pre intervention become 96% Post intervention. Practice of Nurses about incubator care pre59.7% becomes 71.1% which consider significant. Implementation training program about neonatal sepsis was very effective and significantly increases the level of knowledge and practices of Nurses.

Key words: Incubators Care, Neonatal Sepsis, Nursing care

المستخلص

الإنتان الوليدي هو سبب مهم للمرض والوفيات ويعتبر أحد المشاكل المهمة التي لم تحل في جميع أنحاء العالم. نقص المعرفة فيما يتعلق بمكافحة الإنتان الوليدي بين الممرضات يقلل من الامتثال لهذه الممارسات في جميع أنحاء العالم. هدفت الدراسة لمعرفة أثر البرنامج التدريبي على معرفة الممرضات وممارساتهم فيما يتعلق برعاية حديثي الولادة المصابين بالإنتان في مستشفيات أمراض النساء والتوليد، ولاية الخرطوم. تم استخدام دراسة شبه تجريبية في مستشفيات الجيش والشرطة في ولاية الخرطوم. جمعت البيانات البحثية اللازمة باستخدام مقابلة منظمة وقائمة مرجعية قبل وبعد ثلاثة أشهر من التدخل. تم تقديم برنامج تدريبي للممرضات حول الإنتان الوليدي الذي تضمنه (التعريف والأسباب والعلاج والرعاية التمريضية ورعاية الحاضنة) تم تحليل البيانات من قبل الإصدار 20 وصفي SPSS ، واستخدمت الإحصائية الاستدلالية (T.test و chi squared) بقيمة P. كبيرة >0.05 كان المشاركون جميعا من الإناث وكانت الفئة العمرية الأكثر شيوعا بين 20-30 عاما، 35 (74.5%) مع متوسط 27.7 ± 5.3 . معظمهم من حملت بكالوريوس (85.1%) . (التمريض وتجاربهم تختلف ، ولكن الأكثر شيوعا كانت بين 1-5 سنوات و 6-10 سنوات 15) (32%) و 14 (29.8%) على التوالي. كان مستوى المعرفة العام للممرضات فيما يتعلق بتعفن الدم الوليدي 94% في مرحلة ما قبل التدخل أصبح 96% بعد التدخل. ممارسة الممرضات حول رعاية الحاضنات قبل 59.7% تصبح 71.1% التي تعتبر كبيرة. كان برنامج التدريب على التنفيذ حول الإنتان الوليدي فعالا جدا ويزيد بشكل كبير من مستوى المعرفة والممارسات لدى الممرضات.

Introduction

Sepsis remain a leading cause of mortality and morbidity especially during the first five days of life and in low and middle- income countries (LMIC)(Seale2014). According to the world health organization (WHO) four million newborn children die each year during the first four weeks of their lives. of these, 75% die prematurely during the first week of life(Versporten2016) (Neil, David2016) the majors causes of neonatal deaths globally were estimated to be infection (35%), preterm births (28%), intrapartum related complication (24%) and asphyxia (23%), sepsis is the commonest cause of neonatal mortality and is probably responsible for 30-50% of the total neonatal deaths each year in developing countries(Mary2015). **Worldwide:** Incidence of neonatal sepsis is a significant cause of morbidity and mortality of hospitalized newborns and premature infants, sepsis accounts for 15% of neonatal deaths (UNICEF, 2015). **SUDAN:** The prevalence of neonatal sepsis is 17.5% and the mortality is 14.5%.this study was conducted to determine the microbiological profile and antibiogram of neonatal sepsis at Omdurman maternity hospital (Kheir, KheirR.A2014).

Definition

Neonatal sepsis are infections of the neonate acquired during prenatal development or in the first four weeks of life (Neonatal period). (Mary2015).neonatal infections may be contracted by mother to child transmission in the birth canal during child birth, or contracted after birth. (Unicef2015).some neonatal infections are apparent soon after delivery, while others may develop in the postnatal period. Some neonatal infections such as HIV,HIB do not become apparent until much later.

There is a higher risk of infection for preterm or low birth weight neonates.

Causes

Bacteria: such as Group streptococcus are typically identify as the causes of the majority of early-onset infections in the neonate. (Mac Donal, Mhairi2015)

Viruses': Such as Human Immunodeficiency Virus type1 (HIV) infection can occur during labor and delivery, in utero through mother to Child transmission or postnatal by way of breastfeeding. (Polin2014)

Risk factors: Prematurity, meconium aspiration, low birth weight, postpartum endometritis, premature rapture of membranes, vaginal discharge, maternal urinary tract infection

Diagnosis

Samples are obtained from urine, blood or cerebrospinal fluid. Diagnosis of infection can also be aided by the use of more nonspecific tests such as determining the total white blood cell count, cytokine level and other blood tests and signs. (Fanaroff2013)

Management

WHO recommends ampicillin or penicillin, cloxacillin if staphylococcal infection is suspected plus gentamycin for empiric treatment of neonates with suspected clinical sepsis; when referral is not possible, once daily gentamycin plus oral amoxicillin may be used. It's known, however, that

In many countries agents with a broader spectrum, such as third-generation cephalosporin are commonly used to treat neonatal and infant



sepsis. (Neil, David2016). Antiviral medications used include acyclovir, penciclovir; valacyclovir and famciclovir. Only very small amounts of the drug can be detected in the fetus.

Justification

The neonatal sepsis is life threaten disease in the world widespread in Africa, it has high mortality and morbidity rate for infant, to reduce incidence rate should be safe care provide to sepsis, and early detection methods such as, increase awareness, immunization and good nursing care.

Objectives

The general objective of the study is to Know the effect of the Training Program on nurses' knowledge and Practice regarding care of neonate with sepsis in Obstetrics and Gynecology hospitals, Khartoum State, Sudan (2019-2021).

Materials and Methods

Study design

This study is an interventional hospital based study aimed at assessing the effect of the Training Program on nurses' knowledge, and practices regarding nursing care of neonatal with sepsis in Obstetrics and Gynecology hospital, Khartoum State, Sudan, which include 47 participants working in NICU.

Study area This study was conducted in police and military hospitals in neonatal unit at Khartoum state, Sudan.

Study population includes: 47 nurses working in neonatal intensive care unit.



Data collection

A questionnaire, sheet was designed by the researcher in English language included nurses' socio-demographic characteristics such as age, qualifications, years of experience then nurses' knowledge and practice before and after training program. An observational checklist was developed by the researcher to observe the actual nurses' performance before and after the training program. It included all nursing care and procedures needed for the care of neonatal with sepsis.

Phases of the study: Phases of intervention

Pre intervention phase

- Permission was taken from managers and directors of the two hospitals for data collection through official letters.
- A Pilot study was done on a sample of 47 nurses in an effort to test the validity and feasibility of the questionnaire instrument.
- A questionnaire was distributed for each available nurse to fill it within 45 – 60 minutes under the researcher guidance.
- Each nurse had observed for her clinical skills when caring of neonatal with sepsis.

Program implementation: The program was designed to equip the nurses with essential information and practice regarding care of neonatal with sepsis it included (definition, causes, signs and symptoms, signs of infection ,risk factors diagnosis treatment).

Post intervention program: A posttest was made using a questionnaire, to evaluate the effect of program on nurses' knowledge regarding nursing care of neonatal with hypothermia. A certificate and copies of the program was given to all participants. All nurses were observed by the researcher during their performance using and observation checklists regarding neonatal with sepsis.

Data analysis

The data were coded, entered, and analyzed using the statistical package for social science (SPSS) so as to show the result of the hypothesis of the study by using frequencies and percentages in tables, also P.value, Chi-square test to find the statically significant.

Results

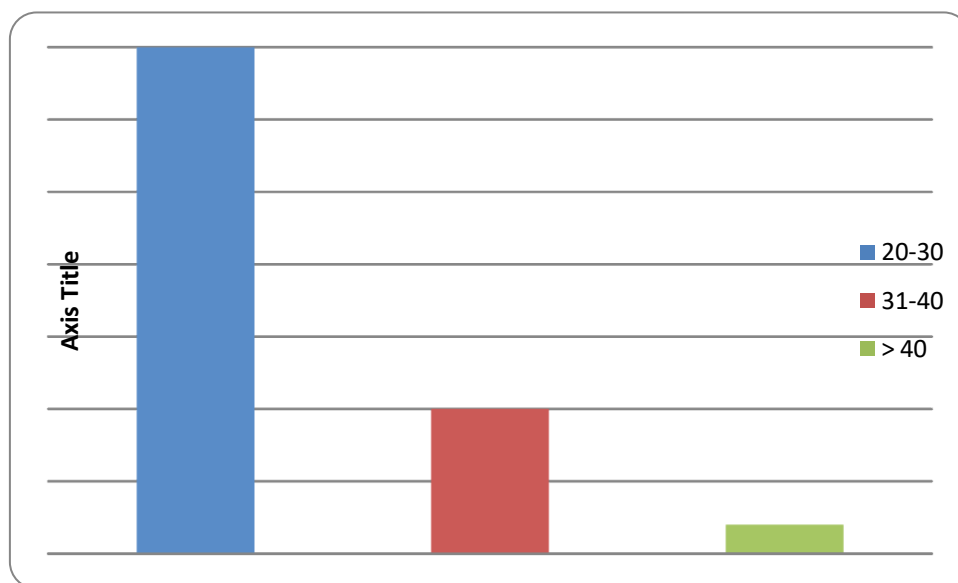
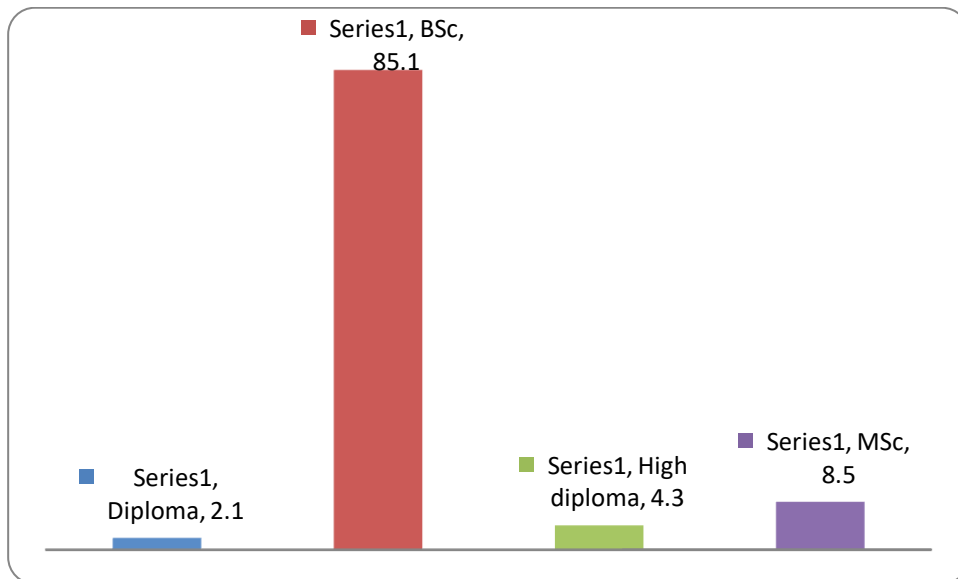


Fig.1 Distribution of the study sample According to their Age (Years)
N=47

This figure shows that the highest of the study sample age (35%) has age between 20-30years



- Fig. 2 Distribution of the Study Sample According to Educational (N=47)

This figure shows that the majority of educational level of the study sample (85.1%) is Bachelor

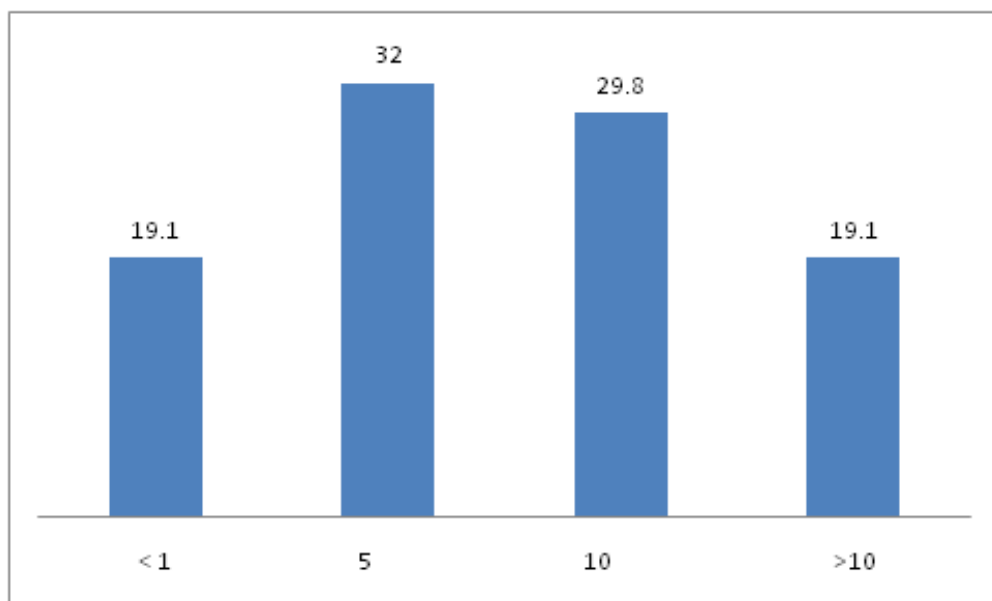


Fig.3 Distribution of the Study Sample According to Years of Experience (N=47)

This figure shows that the highest majority of the study sample (32%) has 1-5 years of experience

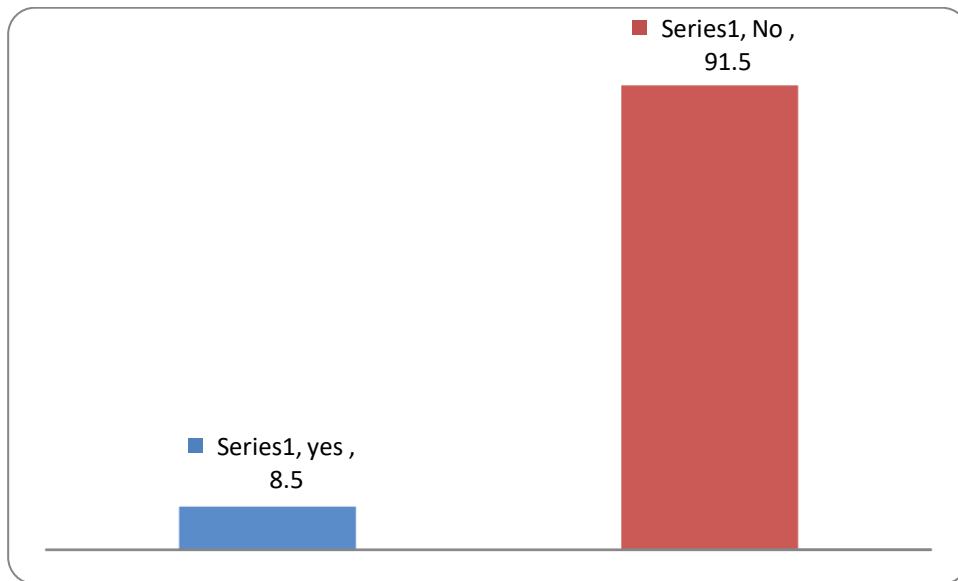


Fig.4 Distribution of the Study Sample According to their Attending of Training Program N=47

This figure reported that they did not attend any training program regarding neonatal sepsis

Table (1) Knowledge of Nurses about Babies with sepsis N=47

V a r i a b l e s	P R E		P O S T		p. value
	F	%	F	%	
Knowledge of nurses about definition of sepsis	37	78.7	41	87.2	0.272
Knowledge of nurses about Causes of sepsis	47	100.0	47	100.0	1
Knowledge of nurses about diagnosis of sepsis	45	95.7	46	97.9	0.557
Knowledge of nurses about Treatment of sepsis	46	97.9	47	100.0	0.315
Knowledge of nurses about Prognosis of sepsis	46	97.9	46	97.9	1
A v e r a g e		94%		96.6%	0.62

Table (2) Nurses practices about incubator care (n=47)

v a r i a b l e s	P R E		P O S T		p. value
	F	%	F	%	
Nurses practice about check temperature of incubator and if need to change water	42	89.4	43	91.5	0.72
Nurses practice about incubator wipe down daily by using minimal soap/ water, don't use alcohol	40	80.9	40	85.1	-
Nurses practice about incubator change every 7 days	9	19.1	10	21.3	0.79
Nurses practice about all inserts are removed and thoroughly wash and dry	11	23.4	16	34	0.25
Nurses practice about change filters every three month(label to indicate the due date)	13	27.7	17	36.2	0.37
Nurses practice about check incubator temperature is 35degree	35	74.5	40	85.1	0.19
Nurses practice about adjust the incubator temperature no more or less than 0.5degree	37	78.7	41	87.2	0.27
Nurses practice about document of temperature 4-6hours as condition dictates	25	53.2	41	87.2	0.00
Nurses practice about use of humidification utilized for incubator care of preterm babies	11	23.4	23	48.9	0.009
Nurses practice about ensure alarms self-test has been completed (automatic)	7	14.9	20	42.6	0.003
Nurses practice about position away from draughts or direct sunlight	42	89.4	45	95.7	0.079
Nurses practice about humidifier chamber must be emptied and cleaned daily , fill with fresh distilled water	35	74.5	42	89.4	0.05
Nurses practice about use of glacial acetic acid 1-2ml or vinegar can be added to water in the humidifier to prevent bacterial growth	42	89.4	42	89.4	-
Nurses practice about sensory stimuli like light and pain should be kept to the minimal	41	87.2	42	89.4	0.74
Nurses practice about check alarm ,don't ignore any alarm	27	57.4	33	70.2	0.19
Nurses practice about keep the water pans filled , be sure to completely replace the water every 1-2 week	34	72.3	40	85.1	0.08
A v e r a g e		59.7%		71,1%	0.35

Discussion

The demographic characteristic of the current study show that the majority of participants(74.5%)are between 20-30years also disagree with study conducted in Yemen which showed that only (83%) have diploma



degree, this is considered as a very serious indicator in Yemen that most of the (Bash2016). staff are not qualified a academically enough to bear their responsibilities as required . regarding the experience the study show that one third of the participant had (1-5years) is (32%).this is disagree with study conducted in (2015),which aims to verify the seriousness of nursing or the discovery of association with respect to nursing care (age, nursing education, and experience in intensive care unit)that show the majority of samples in years of experience group to the study sample where within (under 2 years)in ratio(50%) and (2-5years)in ratio(40%).(Mansi2015).about the training courses, it found that (91.5%)of nurses don't have any training courses in neonatal care. this result is congruent with the result reported by a study in Sudan in which(56%) of nurses don't have any training course in neonatal care units. (Wibside2016).regarding nurses knowledge and practice about incubator care there are improvement in practice(59.7%) pre become(71.1%)post this is disagree with study conducted in (2015) on "effectiveness of incubator care among nurses working in NICU the result show that 46% of them in adequate knowledge and 76%of them had negative attitude and 80% of them had poor practice ,the study concluded that nurses working in NICU knowledge ,attitude and practice was in adequate(angel 2015).regarding nurses knowledge about sepsis this study show that is little improve in knowledge (94%)pre become 96.6% this is disagree with study done in (2013) in NICU at Elminia university and general hospital .this study show that significant progress in nurses knowledge and practices in neonatal sepsis. Concluded that, by the



implementation of program there was remarkable improvement of nurse's knowledge and practice hospitals (abolwafa 2013).

Conclusion

Based on the findings of the present study, it was concluded that: training program are effective and significantly increase the knowledge and improve the practice of the nurses regarding neonatal sepsis.

Recommendation

Based on the study findings and conclusion, the following recommendations are require Based on the study findings and conclusion, the following recommendations are required to be implemented:

1. Develop educational training programs for meeting actual educational need assessment of nurses dealing with neonates in intensive care unit
2. To conduct further studies to assess nurse's knowledge and practice in management of neonates with sepsis

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Determination of Elements in Soil and Croups (Wheat and Sun flower) Using X-MET 5000

Fawaz Taha Omer¹, Alfadil Mahmoud Yousif² & Fawzi Mohamed Ahmed³

¹Department of Physic, Faculty of Education, University of Kassala, Sudan

²Department of Physic, Faculty of Education, University of Albutana, Sudan

³Department of chemistry, Academy of Healthy Since of Blue Nile State, Sudan

Corresponding email: Fawaztaha2018@gmail.com

Abstract

This study aimed to measure the concentration of the elements in soil and crops (Wheat and Sun flower) in Domestic of south Al Gezira. Collected 6 samples of soil and crop full and crash and analyzed by using X-MET 5000. Fe rate between (44-45)(90-92) per-cent for crop and soil by following and for W rate between (5-7) (50-53). Cu rate 0 (1-5) Mn rate 0 (0-1). Ni, V and Cr rate between 0 (0-1) and Mo rate in soil 1% Study Recommended to do more research in this field to measure the concentration of elements in Sudan.

Keywords: X-MET 5000, Elements, Crops, Soil

المستخلص

هدفت هذه الدراسة إلى قياس تركيز العناصر الموجودة في التربة والمحاصيل بجنوب الجزيرة. جمعت 6 عينات من التربة والمحاصيل (قمح و زهرة شمس) تم قياس العينات السابقة كحبوب كاملة ومسحونة وتم تحليلها باستخدام جهاز X-MET 5000 وكانت نسبة الحديد تتراوح بين (44-45) (90-92) بالمائة بالنسبة للمحاصيل والتربة علي التوالي ونسبة التنجستان تتراوح بين (5-7)(50-53) ونسبة النحاس في المحاصيل ضئيلة جدا تؤول إلي الصفر وفي التربة(1-5) ونسبة المنجنيز في المحاصيل ضئيلة جدا تؤول إلي الصفر وفي التربة (0-1) وكانت نسبة كل من النيكل والفانديوم والكروم في المحاصيل والتربة ضئيلة جدا تؤول إلي الصفر بينما ينعدم الموليبدنم في المحاصيل وفي التربة 1%. أوصت هذه الدراسة بالمزيد من البحوث في هذا المجال لقياس تركيز العناصر في مناطق مختلفة في السودان.



Introduction

Matter has distinct levels of structure. For example, atoms, once considered the ultimate building blocks, are themselves composed of nuclei and electrons. The nucleus, in turn, consists of protons and neutrons, which we now believe are made of quarks and gluons. Gaining an understanding of the fundamental structure of matter has not been an easy achievement, primarily because the dimensions of the constituents are so small. For example, the typical size of an atom is about 10^{-8} cm, the average nucleus is about 10^{-12} cm in diameter, neutrons and protons have radii of about 10^{-13} cm, while electrons and quarks are believed to be without structure down to distances of at least 10^{-16} cm. The study of the structure of matter presents formidable challenges both experimentally and theoretically, simply because we are dealing with the sub-microscopic domain, where much of our classical intuition regarding the behavior of objects fails us. Experimental investigations of atomic spectra provided our first insights into atomic structure. These studies ultimately led to the birth of quantum mechanics, which beautifully explained, both qualitatively and quantitatively, not only the observed spectra and the structure of the atom, but also clarified the nature of chemical bonding, and a host of phenomena in condensed matter. The remarkable success of quantum theory in explaining atomic phenomena was mainly due to two reasons. First, the interaction responsible for holding the atom together is the long-ranged electromagnetic force, whose properties were well understood in the classical domain, and whose principles carried over quite readily to the quantum regime.



Second, the strength of the electromagnetic coupling is weak enough so that the properties of even complex atomic systems can be estimated reliably using approximations based on perturbative quantum mechanical calculations. (Azab, 2012)

Study problems

Study the concentration of some elements in the soil and the crops in Domestic of South Al Gezira.

Study Objectives

- Measure the concentration of these elements in the sample.
- Compare the concentration in the crops and the soil.
- Improve the Agricultures in the aria.

Study methodology

The study will use a test to 6 samples using X-ray method.

History of X-MET 5000 in Archaeology

The physical occurrence in the X-MET 5000 process has been termed as a resulting fluorescence emission of x-radiation from the characteristic elements present in a specimen that is exposed to an incident x-radiation beam (Lachance and Fernand,1995). Detection systems take advantage of the fact that emitted radiation has lower energy compared to the incident source and since the emitted radiation is characteristic to specific electron transitions of individual elements, it can be utilized in detecting the chemical composition of specimens (Dasary *et al.*, 2011).



Fig (1) X-MET 5000

X-rays were first discovered by the German physicist Wilhelm K. Rontgen (1845–1923) for which he won the Nobel Prize in 1901 . While X-rays have been used for commercial elemental analysis since the 1950s, X-ray spectroscopy is much older than that, dating back to 1909 when Charles G. Barkla found a connection between X-rays radiating from a sample and the atomic weight of the sample. In 1913, Henry G. J. Moseley helped number the elements with the use of X-rays, by observing that the K line transitions in an X-ray spectrum moved the same amount each time the atomic number increased by one. He is credited with the revision of the periodic tables, which were based on increasing atomic weight, to periodic tables based on atomic number. He later laid the foundation (Steven, 2011).

Definition

X-MET 5000 analysis is a powerful analytical tool for the spectrochemical determination of almost all the elements present in a sample. X-MET 5000 radiation is induced when photons of sufficiently high energy, emitted from an X-ray source, impinge on a material. These primary X-rays undergo interaction processes with the analytic atoms.



High-energy photons induce ionization of inner shell electrons by the photoelectric effect and thus electron vacancies in inner shells (K, L, M, ...) are created. The prompt transition of outer shell electrons into these vacancies within some 100 fs can cause the emission of characteristic fluorescence radiation. Not all transitions from outer shells or sub shells are allowed, only those obeying the selection rules for electric dipole radiation. The creation of a vacancy in a particular shell results in a cascade of electron transitions, all correlated with the emission of photons with a well defined energy corresponding to the difference in energy between the atomic shells involved. The family of characteristic X-rays from each element including all transitions allows the identification of the element. Next to this radioactive form of relaxation, a competing process can take place: the emission of Auger electrons. Both processes have Z-dependent probabilities that are complementary: the Auger yield is high for light elements and the fluorescence yield is high for heavy elements. These techniques can be used for analyzing rocks, metals, ceramics, and other materials. Handling of samples is greatly simplified by the open-air nature of the instrument used for X-MET 5000 studies. However, operation outside a vacuum chamber has the disadvantage of decreased sensitivity to light elements (H.R. Verma , 2007).

Working Principle

The working principle of X-MET 5000 analysis is the measurement of wavelength or energy and intensity of the characteristic X-ray photons emitted from the sample. This allows the identification of



the elements present in the analyse and the determination of their mass or concentration. All the information for the analysis is stored in the measured spectrum, which is a line spectrum with all characteristic lines superimposed above a certain fluctuating background. Other interaction processes, mainly the elastic and inelastic scattering of the primary radiation on sample and substrate, induce the background.

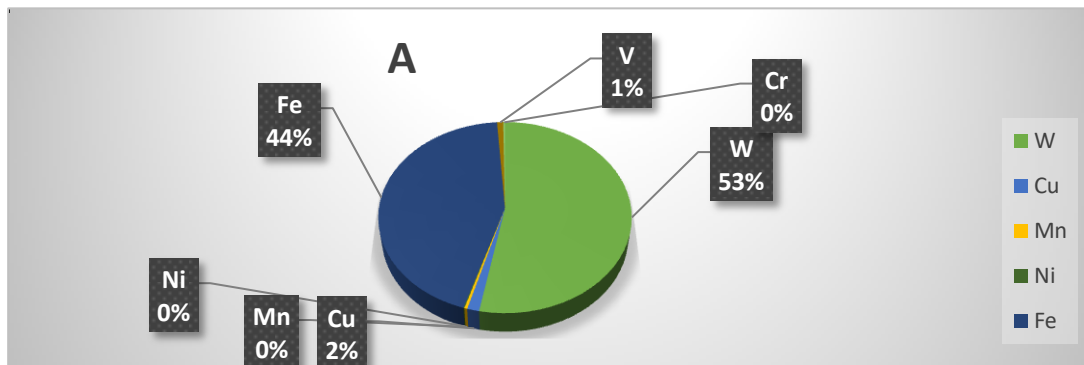
Circumstance of Study

The sample was demonstrated to X-MET 5000 for six second and the experiment proceeding three time and then we calculating the average from the result.

Result and Discussion

Table (3.1): El Remitab Office (60) East Magpool sample of Full Wheat

Sample. No	Element	Concentration
1	W	45.74
2	Cu	1.21
3	Mn	0.29
4	Ni	0.11
5	Fe	38.02
6	V	0.75
7	Cr	0.21



Fig(3.1): sample of Full Wheat

Table (3.2): El Remitab Office (60) East Magpool sample of crash Wheat

Sample. No	Element	Concentration
1	W	45.59
2	Cu	1.09
3	V	0.32
4	Fe	38.22
5	Cr	0.57
6	Mn	0.14

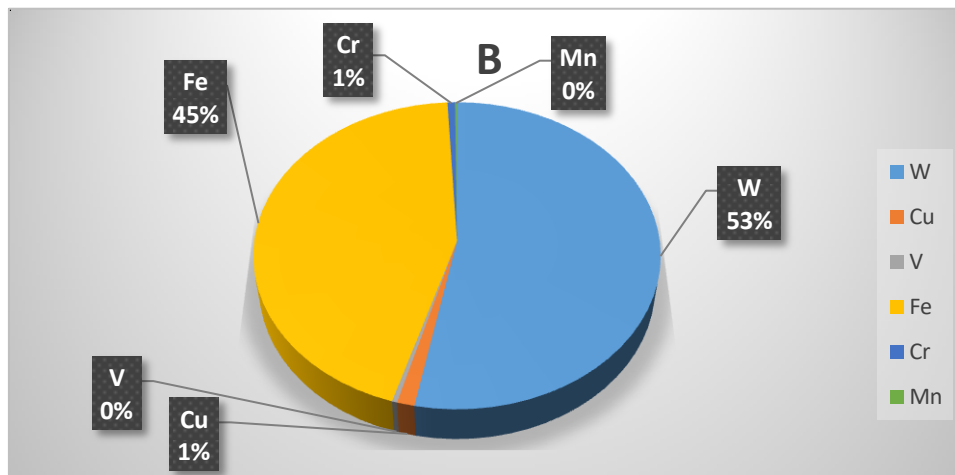


Fig (3.2) sample of Crash Wheat

Table (3.3) El Remitab Office (60) East Magpool soil sample

Sample. No	Element	Concentration
1	Fe	89.86
2	Mn	1.21
3	V	0.76
4	Cu	0.20
5	W	6.48
6	Mo	1.18
7	Ni	0.30
8	Cr	0.02
Sample. No	Element	Concentration
1	Fe	89.86
2	Mn	1.21
3	V	0.76
4	Cu	0.20
5	W	6.48
6	Mo	1.18
7	Ni	0.30
8	Cr	0.02

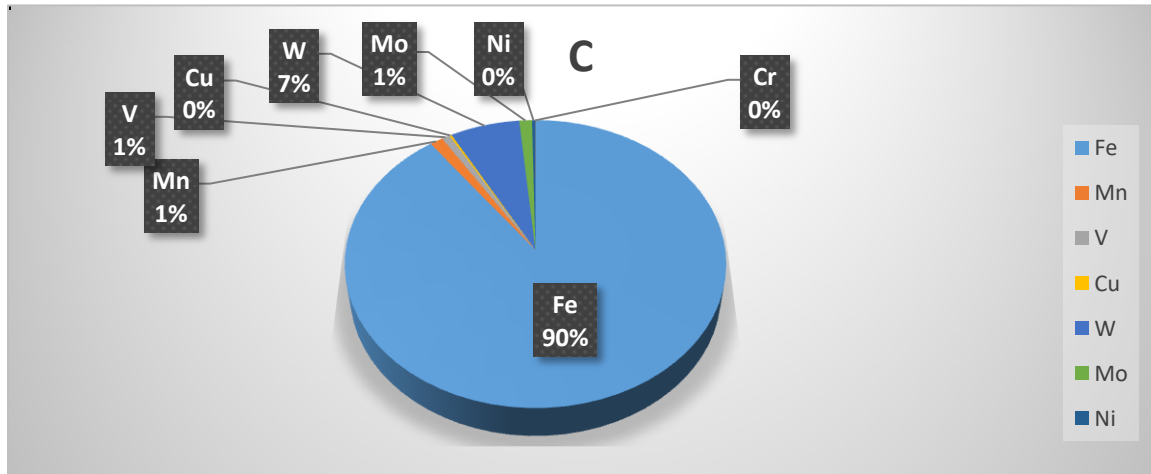


Fig (3.3) soil sample

Table (3.4) El Remitab Office (30) East Shamoon sample of sun flower

Sample. No	Element	Concentration
1	W	43.52
2	Cu	3.66
3	Cr	0.22
4	Fe	38.55
5	V	0.62
6	Ni	0.01

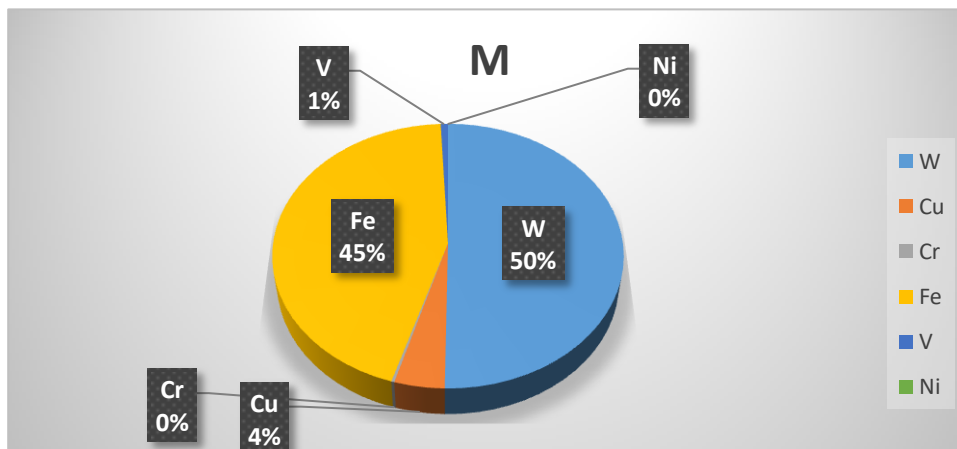


Fig (3.4) sample of Sun flower

Table (3.5) El Remitab Office (30) East Shamoon sample of crash sun flower

Sample. No	Element	Concentration
1	W	42.51
2	Cu	4.11
3	Cr	0.27
4	Ni	0.04
5	Fe	38.29
6	V	0.36
7	Mn	0.09

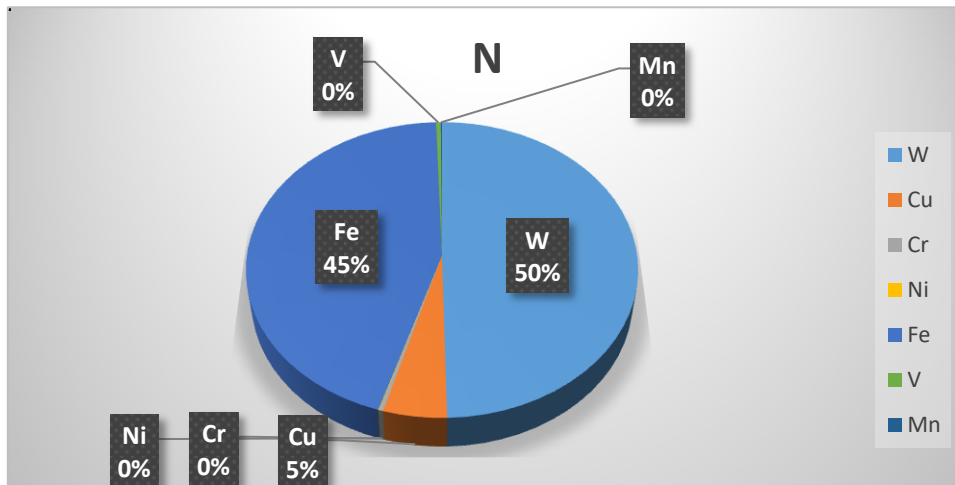


Fig (3.5) sample of Crash Sun flower

Table (3.6) El Remitab Office (30) East Shmoon soil sample

Sample. No	Element	Concentration
1	Fe	91.66
2	Mo	0.97
3	V	0.69
4	Ni	0.23
5	W	4.93
6	Mn	0.82
7	Cu	0.32
8	Cr	0.16

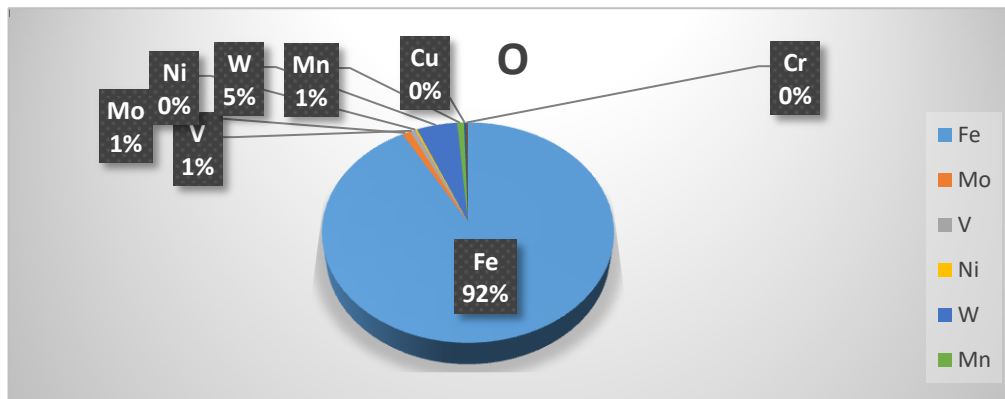


Fig (3.6) soil sample

Discussion

- **Firstly** when samples of Wheat com form EL Remitab Office (60) East Magpool were analysed by X-MET 5000 in table (3.1), figure (3.1). This figure shows that except (Fe 44%- W 53% - Cu 2% - V 1%),but in Table (3.2), fig (3.2). Typical graph of sample (Crash Wheat) purchased from local (EL Remitab Office (60) East Magpool). Except (Fe 45% -W 53% - Cu 1%).And in table (3.3), fig (3.3). Show that the soil sample to give from same local. Except (Fe 90% - W 7% - Mn 1% - Mo 1% - V 1%). (Fe) was decreased from sample (3.1), sample (3.2) than sample (3.3) by rate {44% for the sample (1) and 45% for the sample (2)} this significant change it may be explained by existence of bio effect of growth.

- **Secondly** when samples of Sun flower com form EL Remitab Office (30) East Shamoon were analysed by X-MET 5000 in table (3.4), figure (3.4). This figure shows that except (Fe 45% - W 50% - Cu 4% - V 1%), but in table (3.5), fig (3.5). Typical graph of (EL Remitab Office (30) East Shamoon) sample (Crash Sun flower) purchased from local (30). Except (Fe 45% - W 50% - Cu 5%). And in table (3.6) fig (3.6). show that the soil sample to give from same local. Except (Fe 92% - W 7% - Mn 1% - Mo 1% - V 1%). (Fe) was decreased from sample (1), sample (2) and sample (3) by rate {47% for the sample (1) and 47% for the sample (2)} this significant change it may be explained by existence of bio effect of growth.

Conclusion

Six different samples of soil and crop were analyzed using X-MET 5000. The comparison was done between these samples, a great differences were found between this samples. We found that the total components of (W,Cu,Cr,Fe,V,Ni) samples are the best compared to the other samples in terms of the elements abundance. Measurements also showed some elements are more than other in soil Iron was the highest elements in the soil in the study area by concentration 92% in AL Remitab Office (30) East Shamoon.

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Basic Pharmacology of Losartan

Review Article

Muhanad S. Saad¹, Abdelwhab Hassan Mohamed², Mazin S. Abdalla Mohamed³ and
Fath Elrahman A. Idris⁴

¹Department of physiology, Elrazi University, Sudan

²Department of pharmacology, Faculty of pharmacy, National Ribat University, Sudan

³Department of physiology, Napata College, Sudan

⁴Department of pharmacology, Faculty of pharmacy, Omdurman Islamic University, Sudan

Corresponding email : Muhanadsaad08@gmail.com

Abstract

Renin angiotensin system has been for a long time a focus of researchers from all over the scientific community and in particular physiology and pharmacology. Angiotensin-II is a potent vasoconstrictor; it raises the total peripheral resistance and hence; blood pressure. Angiotensin-II also affects the remodeling of the heart. Losartan was developed to block the receptor of Angiotensin-II, lowering the elevated blood pressure and a positively affect the hemodynamics of the heart.

Key words: Losartan, Angiotensin-II, ACE inhibitor, and RAS

Introduction

Regarding the systemic renin-angiotensin system (RAS), the primary substrate, angiotensinogen, is mainly synthesized by the hepatocytes and then released into the blood. The enzyme Rennin (a product of the kidney) acts on angiotensinogen to produce angiotensin (Ang I). This product is then converted into the (Ang II) by an enzyme named angiotensin converting enzyme (ACE), which is prominently encountered in the pulmonary vascular endothelium. Angiotensin II is considered to be the most active peptide of the system and it has subtypes



of receptors named (Ang II) subtype 1 (AT1) and subtype 2 (AT2) (Fitzsimons JT., 1998, and Paul M et al ., 2006). A very important cause of death globally is still to this day attributed to the disease of the cardiovascular system (World Health Organization 2017, and Johnson et al 2017) Uncontrolled hypertension leads to chronic kidney disease (Ashish Verma et al., 2013). Taking this into account, there is a nagging need to take actions towards the prevention of cardiovascular ailments, the development of chronic kidney disease; these actions should rely of the lifestyle changes and drug therapy(Volpe M et al., 2013) Results from clinical trials targeted the prevention and treatment of cardiovascular diseases with blocking Ang II receptors by pharmacological means (Smith DH., 2008). Since Losartan is considered as a selective AT1 receptor antagonist, which had an unmistakable attention in experimental and medical studies (Ramasubbu K., et al 2007), it is important to focus on the pharmacokinetic, pharmacodynamics, and adverse effect of the losartan as a drug of use.

Losartan pharmacology

Losartan is a 4-chloro-5-hydroxymethylimidazole derivative that is a potent and highly selective angiotensin II receptor antagonist (Stearns RA. et al., 1995). Losartan is a competitive antagonist that causes a parallel rightward shift of the concentration-contractile response curve to angiotensin-II without depression of the maximal presser response (Sica DA et al., 2005).

Pharmacokinetics of Losartan

Sica DA et al., (2005) & Almajed AR et al., (2015) demonstrated the pharmacokinetics of losartan . Losartan is the first orally available



angiotensin receptor antagonist without agonist properties, rapidly absorbed with the bioavailability of 33%, and reaching its maximum concentration within 1-2 hours after an oral administration. About 14% of the losartan dose is converted to its pharmacological active metabolite E3174 (Paul M et al., 2006). Potency of E3174 is about 10-40 times more than losartan with estimated half-life range from 6-9 hours (Sica DA et al., 2005. and Almajed AR et al., 2015). The pharmacokinetics of losartan and its active metabolite E3174 are linear, dose proportional, and do not substantially change with repetitive administration (Sica DA et al., 2005). Losartan and its E3174 metabolite have a relatively low volume of distribution 34.4 ± 17.9 & 10.3 ± 1.1 L respectively (Sica DA et al., 2005). The major metabolic pathway for losartan via cytochrome P450, and the total plasma clearance of losartan and its active metabolite E3174 is approximately 600 & 50 ml/min respectively (Sica DA et al., 2005).

Pharmacodynamics of losartan

Suzuki et al., (2001) showed the effects of losartan on the cardiovascular system. Losartan administration causes a reduction in the mean arterial blood pressure, pulmonary artery pressure, and peripheral resistance, In addition to an increase in the stroke volume after treatment with losartan (Suzuki J et al., 2001) The effect of losartan on blood pressure also demonstrated by Koprdoва et al., (2009) who reported that, treatment with losartan cause a decrease in the blood pressure and heart weight to body weight ratio. Moreover rats treated with Losartan caused an increase in the *alpha* cardiac myosin heavy chain expression, decrease in the beta cardiac myosin heavy chain expression (Babick A et al., 2012, and Zhang ML et al., 2003) Also treatment with Losartan reduces hepatic



venous pressure in patients with portal hypertension, and prevent doubling of serum creatinine concentration in patient with diabetic nephropathy (Schneider AW et al., 1999, and Brenner BM et al., 2001).

Adverse effects of Losartan

Cough and Rectus sheath hematoma Spontaneous rectus sheath hematoma is a rare condition and 29% of such cases are caused by cough. Owing to its rarity, it is clinically unfamiliar and can be easily missed or confused with other acute abdominal conditions. Early diagnosis is necessary to prevent life-threatening complications like haemorrhagic shock and death (Gotham et al., 2016). Here we put the light on a case reported by Gotham et al., (2016). The case was on losartan for hypertension which was started few weeks prior to the onset of cough. Which led to the development of rectus sheath hematoma in the clinical context of being anticoagulated with Rivaroxaban (Gotham et al., 2016). The proposed mechanism of cough with ARBs is via AT1 receptor blockade subsequently activating AT2 receptors due to increased levels of circulating angiotensin II. AT2 receptor activation leads to activation of the bradykinin–prostaglandin– nitrous oxide cascade which causes cough (Godlike P et al., 1998). Hyperkalaemia and ARBs In a similar study showed that treatment with losartan increased the serum potassium concentration. Furthermore demonstrated that the occurrence of high serum potassium levels increased the risk of adverse renal outcomes and counteract the beneficial renoprotective effects of losartan (Y. Miao et al., 2011).

Angioedema

The protection offered by ARBs against adverse effects like angioedema thought to be mediated by kinins may not be absolute. There is considerable evidence for ARB-induced angioedema in literature (Matcher DB et al., 2008, and Toh S et al., 2012) Amongst ARBs, losartan had the highest hazard ratio for angioedema events, considering the fact that losartan is the most widely used ARB (Toh S et al., 2012).

Another concept that may describe the phenomenon behind ARB angioedema is that the ARB molecule exerts a competitive inhibition on the AT1 receptor. Hence, the feedback-induced increase in plasma levels of angiotensin II levels may, per se, activate vascular AT2 receptors that are available and generate bradykinin (Irons BK & Kumar A 2003). This, in turn, may stimulate bradykinin BK receptors in the endothelium and causes vasodilation and angioedema (Audit G et al., 2001, Bas M et al 2007) . Cross-reactivity of angioedema between angiotensin converting enzyme inhibitors (ACEIs) and ARBs could affect visceral organs (Prashanth M. et al., 2014). Meta-analyses recommend cautious use of Losartan or other ARBS after an episode of ACEI-induced head, neck, and visceral angioedema after appropriate counselling (Prashanth M. et al., 2014).

Conclusion

The pharmacologic arsenal has added the drug Losartan in the front line in the fight against hypertension. Losartan is the first orally available drug of the family of receptor blockers (ARBs). Blocking the receptors of angiotensin-II by has demonstrated so many positive effects on the cardiovascular as well as extra cardiac physiology from the reduction of



blood pressure, portal vein pressure to the down regulation of the expression of beta myosin heavy chains. Many side effects caused by ACE inhibitors have disappeared by the use of Losartan, however, incidences of cough, angioedema and electrolyte disturbances have been reported.

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The effect of Black Cumin (*Nigella sativa* L.) and Garlic *Allium sativum* L.) Oils on Dermatophytosis

Tayseer H.A. Mohammed¹, Khalid E.M. Idris² and Elebaid M. Y. Elhaj²

¹Ministry of Education, Gezira state, Sudan

²Department of chemistry, Faculty of Education, University of Gezira, Sudan

Abstract

Ringworm is a fungal infection caused by a type of fungus called dermatophytosis or tinea belonging to a group of mold like skin fungi that live on cells in the outer layer of the skin. Substances derived from the plants remain the basis for a large production of the commercial medication used today for the treatment of heart diseases, high blood pressure, pain, asthma, and other problems. Black cumin (*Nigella sativa* L.), family Ranunculaceae, is a widely distributed annual herbaceous plant. Garlic (*Allium sativum* L.) Has been used worldwide for centuries as a spice, food and folklore medicine to cure and prevent various illnesses.

Keywords: Dermatophytosis, Black cumin, Garlic

Introduction

Skin, hair, nail, and subcutaneous tissues in human and animal are subjected to infection by several organisms, mainly fungi named dermatophytes and cause dermatophytoses (Valeria *et al.*, 1996; Amer *et al.*, 2006). Dermatophytoses are one of the most frequent skin diseases of human, pets and livestock (Tsang *et al.*, 1996).

Tinea or ringworm is the other name for dermatophytosis. Inflammation as ring shape with clear center lesion is the well diagnostic clinical features of dermatophytosis. Few cases were reported about distributed of dermatophytes infection into deep tissues under skin. Symptoms are very limited with itching and odor in some type of tinea. Clinical signs are



various based on the nature of infection and the location on human body (AL-Janabi, 2014).

Black cumin (*Nigella sativa* L.), family *Ranunculaceae*, is a widely distributed annual herbaceous plant. The seeds of black cumin have been used extensively in foods and added as a spice to a variety of Persian foods such as bread, yogurt, pickles, sauces and salads for flavoring (Zargari, 1990). They are also used in Iranian folk and traditional medicines for treating some respiratory, gastrointestinal, rheumatic and inflammatory disorders (Amin, 1991; Nafisy, 1989; Zargari, 1990).

Garlic (*Allium sativum* L.) has been used worldwide for centuries as a spice, food and folklore medicine to cure and prevent various illnesses (Haciseferoğullari *et al.*, 2005). It has numerous health benefits confirmed by numerous studies, which include its ant arthritic, antithrombotic, anticancer and antimicrobial activities (Amagase *et al.*, 2001; Corzo-Martínez *et al.*, 2007). Garlic has also been used to treat acne, ringworm, high blood pressure, gastrointestinal problems as well as asthma (Deresse and Mohammed, 2009; Kumar *et al.*, 2010).

Material and methods

The seeds of Black cumin (*Nigella sativa* L.) and samples of Garlic (*Allium sativum* L.) were obtained from the local market in Al hasaheisa City, Sudan. This study was conducted at Khalawi Elsheikh Taha, located in the village of Sheikh Taha, which there are schools for the memorization of Koran .Located on the Khartoum road and border at north to Aldwynib village, south to Aikoura village and east to Blue Nile, with a population about 350 peoples. Students intended it from different parts of Sudan, they are 400 students and their age between 7 to 23 years.

Preparation of Black Cumin Extract by Hexane

Taken 75 gram of fresh Garlic and placed in filter paper which placed into extraction thimble and 150 ml of hexane were put in soxhlet extractor, hexane heated to reflux and travels into thimble and soluble compound were transferred into the hexane this practice repeated to several hours.

Preparation of Garlic Extract by Hexane

Taken 75 gram of fresh Garlic and placed in filter paper which placed into extraction thimble and 150 ml of hexane were put in soxhlet extractor, hexane heated to reflux and travels into thimble and soluble compound were transferred into the hexane this practice repeated to several hours.

Results and Discussion

Subjects by Age shown in Table (1), this study targeted the age of students between 8 to 19 years and the percentage of students, ages from 8 to 13 years was 68 % and the percentage of students ages from 14 to19 was32%. This means that fungus is more prevalent in the younger group.

Table 1: The Distribution of the Study Subjects by Age

Age group	Frequency
8-13	68
14-19	32

The present study was conducted to compare the biological activity of essential oils Black cumin against the fungus dermatophytosis infected human skin cells.

The antifungal activities of black cumin oil on *Dermatophytosis* infected human skin cells at different visits were observed on the growth of *Dermatophytosis* shown in Table (4.1), from the results the black cumin oil was inhibiting growth of fungus completely.

Table 2: The effect of Derematophytosis of the Black Cumin oil

Name	Area of injury	Visit (1)	Visit (2)	Visit (3)	Visit (4)	Visit(5)	Visit (6)
Sample 1	Head	4 cm	3 cm	2 cm	1 cm	0.00	0.00
Sample 2	Head	3 cm	2 cm	1 cm	0.5 cm	0.00	0.00
Sample 3	Head	3 cm	2 cm	1 cm	0.00	0.00	0.00
Sample 4	Head	3 cm	1 cm	0.00	0.00	0.00	0.00
Sample 5	Head	4 cm	3 cm	3 cm	1 cm	0.00	0.00
Sample 6	Head	4 cm	3 cm	2 cm	1 cm	0.00	0.00
Sample 7	Head	3 cm	2 cm	0.5 cm	0.00	0.00	0.00
Sample 8	Head	2 cm	1 cm	0.5 cm	0.00	0.00	0.00
Sample 9	Head	4cm	3cm	2cm	1cm	0.00	0.00
Sample 10	Head	3cm	2cm	1cm	1cm	0.00	0.00
Sample 11	Head	2 cm	1 cm	0.5 cm	0.5 cm	0.00	0.00
Sample 12	Head	3 cm	2 cm	1 cm	0.5 cm	0.5 cm	0.00
Sample 13	Head	4 cm	2 cm	1 cm	0.5 cm	0.00	0.00
Sample 14	Head	2 cm	1 cm	0.5 cm	0.00	0.00	0.00
Sample 15	Head	3cm	2cm	1cm	0.00	0.00	0.00
Sample 16	Head	4cm	2cm	1cm	0.5 cm	0.00	0.00
Sample 17	Head	4 cm	3 cm	2 cm	1 cm	0.00	0.00
Sample 18	Head	3 cm	2 cm	1 cm	0.5 cm	0.00	0.00
Sample 19	Head	4 cm	3 cm	2 cm	1 cm	0.5 cm	0.00
Sample 20	Head	2 cm	1 cm	0.5 cm	0.00	0.00	0.00
Sample 21	Head	3 cm	2 cm	1 cm	0.5 cm	0.5 cm	0.00
Sample 22	Head	5 cm	4 cm	3 cm	1 cm	0.00	0.00
Sample 23	Head	4 cm	2 cm	1 cm	0.5 cm	0.00	0.00
Sample 24	Head	3cm	2cm	1cm	0.00	0.00	0.00
Sample 25	Head	4cm	3cm	1cm	0,5	0.00	0.00
Mean		3.32	2.16	1.22	0.5	0.06	0.00

The present study was conducted to compare the biological activity of essential oils Black cumin against the fungus *Dermatophytosis* infected human skin cells. The antifungal activities of Garlic oil at different visits were observed on the growth of *Dermatophytosis* shown in Table (4.2), from the results the Garlic oil was inhibiting growth of fungus completely.

Table 3: The effect of the Derematophytosis of the Garlic oil

Name	Area Of injury	Visit 1	Visit 2	Visit 3	Visit 4	Visit 5	Visit 6
Sample 1	Skin	2.5cm	2cm	1cm	0,5cm	0.00	0.00
Sample 2	Skin	3cm	2cm	1cm	0.5	0.00	0.00
Sample 3	Skin	2cm	1cm	0.5	0.00	0.00	0.00
Sample 4	Skin	3cm	2cm	1cm	0.5cm	0.00	0.00
Sample 5	Skin	3cm	2cm	1cm	0.00	0.00	0.00
Sample 6	Skin	2cm	1.5cm	1cm	0.00	0.00	0.00
Sample 7	Skin	3cm	2cm	1cm	0.5cm	0.00	0.00
Sample 8	Skin	4cm	3cm	2cm	0.5cm	0.00	0.00
Sample 9	Skin	3cm	2cm	1cm	0.5cm	0.00	0.00
Sample 10	Skin	4cm	3cm	2cm	0.5cm	0.00	0.00
Sample 11	Skin	2cm	1.5cm	1cm	0.00	0.00	0.00
Sample 12	Skin	2cm	1.5cm	1cm	0.00	0.00	0.00
Sample 13	Skin	3cm	2cm	1cm	0.5cm	0.00	0.00
Sample 14	Skin	3cm	2.5cm	1.5cm	0.5cm	0.00	0.00
Sample 15	Skin	1.5cm	1cm	0.5cm	0.00	0.00	0.00
Sample 16	Skin	2.5cm	2cm	1cm	0.5cm	0.00	0.00
Sample 17	Skin	2cm	1.5cm	1cm	0.00	0.00	0.00
Sample 18	Skin	2cm	1cm	0.5cm	0.00	0.00	0.00
Sample 19	Skin	4cm	3cm	2cm	0.5cm	0.00	0.00
Sample 20	Skin	3cm	2.5cm	1.5cm	0.5cm	0.00	0.00
Sample 21	Skin	2cm	1.5cm	1cm	0.00	0.00	0.00
Sample 22	Skin	3cm	2cm	1cm	0.00	0.00	0.00
Sample 23	Skin	3cm	2cm	1cm	0.5cm	0.00	0.00
Sample 24	Skin	2cm	1.5cm	1cm	0.00	0.00	0.00
Sample 25	Skin	2cm	1.5cm	1cm	0.00	0.00	0.00
Mean		2.64	1.86	1.1	0.28	0.00	0.00



Conclusions

The Garlic oil has antifungal properties and inhibited the growth of *Dermatophytosis* fungus completely after five weeks. The Black cumin oil has also an antifungal properties, it inhibited the growth of fungus completely in the sixth weeks. Compare between two oils tested, the garlic oil gave the best results.

Recommendations

The study recommended investigating of the antibacterial effects of these oils as well as the antimicrobial effects of some other herbal essential oils.

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The effectiveness of the Random Forest algorithm in monitoring abnormal withdrawals to detect credit cards frauds

Hamza Mohamed Hassan¹, Awad Haj Ali Ahmed²

¹Information Technology Department , Al Neelain University, Sudan

²Computer Science Department, Al Neelain University, Sudan

Corresponding email: hamza_kambal@hotmail.com

Abstract

Today, the reach of the Internet and the broad range of options such as e-commerce, online shopping, have gained a great deal of attention. On the other side of the coin, customers are faced with negative benefits due to fraudulent activities. Credit card fraud refers to the physical loss of credit card or loss of sensitive credit card information. The Credit Card Fraud Transaction Detection System is a method used to identify fraudulent transactions that take place every once in a while. Classification techniques are most commonly used for the analysis of predictions. Prediction of the detection of credit card fraud is therefore the main objective of this work. Authors seek to implement the latest data mining techniques dubbed "machine learning techniques", which allows owners and service providers to identify fraud in the credit card and realize whether the purchase is fraudulent or legitimate. Our aim here is to identify fraudulent transactions while eliminating incorrect classifications of fraud. The project consists mainly of four major algorithms and uses anomaly detection as a method to classify fraudulent transactions. In this paper, we have proposed the use of seven classifiers to detect fraudulent credit card transactions. This choice was made by evaluating different methods, including Random Forest, Logistic Regression, Gradient Boosted, Multi Layers Perceptron, Support Vector Machine, Decision Tree and Fuzzy Rule. In this paper, we worked with European credit card fraud dataset. Test and training sets are the two sub-parts of the input data. In terms of precision, recall and F-measurement, the normal and fraud transactions have been predicted on the basis of test and training sets. The performance of the algorithms is measured based on recall, precision and f-measure. Compared to proposed algorithms with two feature selections, we suggest that the Random Forest is best algorithm, and more effective in F-measurement with 86% degree. And genetic algorithms are the best technique for selecting a feature from a dataset, and the Random Forest algorithm outperformed the other proposed algorithms. From the results obtained, it is clear

that Random Forest obtained average Recall scores 86.15% and 84.87% for Genetic Algorithms and Feature Elimination for Features Selection respectively, among all algorithms, indicating the ability of Random Forest correctly detect more than 86% of the suspicious credit card transactions with a low false-negative percentage in Genetic Algorithms and more than 84% in Feature Elimination for Features Selection.

Keywords: Credit Card; Fraud Detection; transactions; Random Forest; Logistic Regression; Gradient Boosted; Support Vector Machine; Fuzzy Rule; Multi Layers Perceptron; Decision Tree.

المستخلص

في هذه الأيام ونظراً للتطور المتسارع في شبكات الإنترنت، إزدادت الحاجة للتعامل مع موقع الدفع الإلكتروني مثل مواقع التسوق عبر الإنترنت ومواقع التجارة الإلكترونية المختلفة ، مما يقود للتعامل عبر بطاقة الإئتمان لإجراء عمليات الدفع اللازمة وبالتالي إكتسبت قدراً كبيراً من الإهتمام. على الجانب الآخر من العملة، يواجه العملاء مزايا سلبية بسبب الأنشطة الإحتيالية، يشير الإحتيال في بطاقة الإئتمان إلى الخسارة المادية لبطاقة الإئتمان أو فقدان معلومات بطاقة الإئتمان الحساسة. نظام الكشف عن عمليات الإحتيال ببطاقة الإئتمان هو طريقة تستخدم لتحديد المعاملات الإحتيالية التي تحدث بين الحين والآخر. تستخدم تقنيات التصنيف بشكل شائع لتحليل التنبؤات. لذلك فإن التنبؤ بالكشف عن الإحتيال في بطاقات الإئتمان هو الهدف الرئيسي لهذا العمل. يسعى المؤلفون إلى تطبيق أحدث تقنيات التنقيب عن البيانات التي يطلق عليها اسم "تقنيات التعلم الآلي"، والتي تسمح للمالكين ومقدمي الخدمات بتحديد الإحتيال في بطاقة الإئتمان وإدراك ما إذا كانت عملية الشراء إحتيالية أو مشروعة. هدفنا هنا هو تحديد المعاملات الإحتيالية مع التخلص من التصنيفات غير الصحيحة للإحتيال. في هذه الورقة، إقترحنا استخدام سبعة مصنفات للكشف عن معاملات بطاقات الإئتمان الإحتيالية. تم إجراء هذا الإختيار من خلال تقييم طرق مختلفة، بما في ذلك الغابة العشوائية (Random Forest)، الإندارد اللوجستي (Logistic Regression)، تعزيز الدعم (Gradient Boosted)، الشبكة متعددة الطبقات (Multi Layers Perceptron) وألة موجه الدعم (Support Vector Machine) وشجرة القرار (Decision Tree) والقاعدة الضبابية (Fuzzy Rule) وذلك بالتطبيق علي طريقتي الخوارزميات الجينية وإختيار الميزة الخلفية لإختيار الميزة. خوارزمية الغابة العشوائية لها أفضلية نسبة لتمييزها في معظم المقاييس وبالأخص مقياس في مقياس F (F-measurement) بنسبة 86.15% ، وذلك عند إختيار كل ميزات مجموعة البيانات وتطبيق إستراتيجية الخوارزميات الجينية لإختيار الميزة. وهذه النسبة توضح أن لخوارزمية الغابة العشوائية إحتمالاً بنسبة أعلي من

86% للتبؤ بمعاملة إحتيالية في معاملات بطاقات الإئتمان (Credit Cards) وأن تكون هذه المعاملة الإحتيالية حقيقية بالفعل.

Introduction

New developments in electronic retail and communications technologies have rendered the credit card the potentially most common payment form for both daily and online purchases; As a result, there is a significant increase in fraud associated with such transactions. Fraud can be defined as false or criminal deception that is intended to give rise to financial or personal gain (Sahin and Duman 2011). On other hand, Credit card generally refers to a card that is assigned to the customer (cardholder), usually allowing them to purchase goods and services within credit limit or withdraw cash in advance (Abhilasha et al., 2019). Card payments are very popular payment method in nowadays. Card payments are quite easy to perform on merchant side by presenting credit card or on internet by announcing credit card details: number, expiring date and security code (Dornadula and Geetha 2019). It is not essential to use the physical card during online payment time, only few information regarding the card is enough (Kolli and Uma Devi 2019). One way by which credit card fraud takes place is by getting access to the stolen credit cards and second is by exploiting the details of the card via online transaction without the knowledge of the genuine card holder. There are growing numbers of new companies all around the world. All of those companies are trying to provide best service quality for their customers (Varmedja et al., 2019). Fraudulent credit card purchases cause businesses and customers a great deal of financial damage per year, and fraudsters are continually attempting to discover innovative technology and ways to participate in fraudulent transactions. The prevention of unauthorized purchases has been a significant factor impacting the growing usage of online payments (Taha and Malebary 2020). Several strategies are developed and applied to tackle the identification of credit card fraud, such as genetic algorithm, artificial neural network, deep learning algorithms, logistic regression, SVM, decision tree and random forest (Khare and Yunus 2018). The purpose of this paper is to analyze various machine learning algorithms, such as Naïve Bayes Classifier (NBC), Isolation Forest and Local Outlier Factor (LOF) in order to determine which algorithm is most suitable for credit card fraud detection. The behavior of these anomalies can be analyzed and a comparison of the final results to verify the best and most appropriate algorithm to detect credit card fraud (Varmedja et al., 2019). The performance of the algorithms is evaluated by the following performance matrices: Accuracy, Recall and Precision. In this paper analysis of the dataset is performed which is taken from Kaggle (Kulkarni et al., 2019). The dataset contains Credit card transactions which are made by customers during September 2013 in Europe. By monitoring the behavior of the



transactions Credit card transactions are characterized into two categories fraudulent and non-fraudulent. The remainder of this paper is structured as follows: Section 2 includes a thorough analysis of credit card fraud, identification of features and performance comparison. Section 3 gives a quick overview of the classification approaches used to build the credit card fraud detection system classification models set out in this paper and proceeds to include descriptions of our methodology. Section 4 presents the findings of the experiments and addresses comparative study. Section 5 concludes the comparative analysis and includes recommendations for possible research areas and sets out directions for future work. The rest of the paper is organized as follows: describes the related work about the credit card system in Section II. Section III provides a quick overview of the classification approaches used to build the classification models of the credit card fraud detection system set out in this paper, and goes on to include descriptions of our methodology. Section IV shows the performance analysis and results. Finally, the study's conclusions are summarized in Section.

According to Varmedja et al., (2019), Kolli and Uma Devi, (2019) k-Nearest neighbors (KNN) and outlier detection techniques can also be efficient in fraud detection. They are proven useful in minimizing false alarm rates and increasing fraud detection rate. KNN algorithm also performed well in experiment for paper (Rezapour, 2019), where the authors tested and compared it with other classical algorithms. Authors Dornadula, and Geetha, (2019) used machine-based learning algorithms to detect fraud and did a comparative analysis of the Random Forest algorithm and the Local Outlier Factor. Authors showed Credit card fraud detection is efficient by both of these algorithms but combining of more than one algorithm will give higher efficiency. John and Naaz (2019) attempted to increase the accuracy of highly imbalanced real-world datasets by ensemble learning methods. Authors using Local Outlier Factor and Isolation Forest for anomaly detection. When analyzed and their final results compared, they showed the highest accuracy rate was given by the Local outlier factor. Authors Varmedja et al., (2019) showed a predictive model that capture the fraudulent transactions with high accuracy using Isolation Forest & Local Outlier factor for detecting outliers that explicitly identifies anomalies and Extreme Gradient Boosting, an ensemble approach for constructing and evaluating the predictive model. The problem is if the dataset increases more it may lead to over fitting problem. Application of Deep Learning concepts perform better analysis and can deliver risk scores in real-time with better accuracy. Ali Shukur and Kurnaz (2019) proposed use of LOF for detection of credit card fraud and implemented in MATLAB technology, and the efficiency of the system is evaluated in terms of the true negative, false positive rate and accuracy of the system over the various neighbors. Accuracy of our proposed solution lies between to 60-69 % for dataset 1

and 96 % for dataset 2 with variation in neighbors. This system can be scale-up for high transactions volumes. Similar research domain was presented by Authors Dhankhad et al., (2018), which designed a model for detecting fraud activity in credit card transactions. Where they have trained and tested datasets using a random forest algorithm and decision trees. The performance of the techniques is measured based on accuracy, sensitivity, and specificity, precision. Results are indicated that the Random Forest Algorithm will provide better results performance with many training data, but speed during testing and application will still suffer. In 2019, Rezapour a study conducted to compare the performance of different unsupervised methods for the detection of credit card fraud. The advantage of mahalanobis methods over the other two methods is that this method does not need to be trained on labeled data and it can identify anomalies based on the minimum covariance. For the future studies both global and local outliers need to be considered for those studies. Modi (2017) applies used neural network Coevolutionary SMOTE, transforming the original features into new ones. Increase the TP number and decrease the FP number to provide improved results. The result of the comparison indicates that precision output in CNN is poor than that of NN, because the ratio of valid transactions defined as fraudulent is more than neural network. Khare, and Yunus (2018) examines and tracks the performance of the Decision Tree, Random Forest, SVM and logistic regression of highly distorted data on credit card fraud. The results thus obtained conclude that the Random Forest has the most accurate and high accuracy of 98.6%. With more training data, the random forest algorithm performs better, but the speed will suffer during testing and application. Incorporating further pre-treatment treatments would also improve. Incorporating further pre-treatment treatments would also improve. The SVM algorithm still suffers from the imbalanced data set issue and needs more pre-processing to deliver better results on the SVM results, but it could have been better if more pre-processing was done on the data.

Methodology

In this research, seven separate algorithms were used to identify credit card fraud and to classify transactions as fraud or valid transactions. Tests are performed and the evaluation of these experiments is carried out using the confusion matrix and we test and determine the effectiveness of the suggested approaches utilizing measures such as: precision, recall and F-measurement. In order to ensure the effectiveness of the algorithms used, the concept of feature selection was applied, where we used two methods for selecting the feature, namely, the method of genetic algorithms for feature selection and the method of Backward Feature Selection. Such algorithms are contrasted to decide which algorithms give better performance and can be modified to identify fraud by credit card traders (Lakshmi and Kavila 2018).

Machine Learning Methods

Machine learning approaches are also used for the evaluation of the credit score system because they make less presumption and have greater analytical precision.

Supervised learning and unsupervised learning

Using a supervised method helps to find a label on past transactions, they tend not to recognize the pattern of fraud that has occurred in the past. While the unsupervised technique helps to find out the transaction class (Dhankhad et al., 2018).

Fraud Detection Classifier

Random Forest Algorithm

Random Forest algorithm is a supervised machine learning based algorithm that combines many decision trees together to achieved an effective outcome. Decision trees are generated by random forest algorithm based on data samples and the best solution is selected by voting (Abinayaa et al., 2020). On the other hand, A random forest is a meta estimator that fits a number of decision tree learners on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting (Taha and Malebary 2020). Ensemble learning is an algorithm where the predictions are made by combining or labeling various models or related models multiple times.

Logistic Regression

Logistic regression is a popular supervised learning classifier that is mostly used in data processing, disease detection and economic prediction. Logistic regression performance will determine the probability of a class. The default logistic regression threshold is 0.5 (Zhang et al., 2019). Logistic Regression Model is a generalized form of Linear Regression Model. It is a very good Discrimination Tool. Logistic Regression is a well-set statistical technique because of predicting binomial or multinomial outcomes. Multinomial Logistic Regression algorithm will generate models when the target field is set with two or more potential values (Sahin and Duman 2011).

Gradient Boosting Machine Learning

GBM is a Gradient boosting framework that uses tree-based learning algorithms. It is designed to be distributed and efficient. Gradient Boosting Machine (GBM) is based on the traditional Gradient Boosted Decision Tree (GBDT)

algorithm, which can increase the training speed of GBDT model without losing its accuracy. The high performance GBM algorithm can manage large volumes of data efficiently, as well as the distributed data processing. It had been developed by Microsoft as an open source project (Taha, and Malebary 2020). GBM uses the Histogram algorithm and the Leaf-wise growth strategy to achieve this effect. The idea of Histogram algorithm is to transform the continuous floating-point eigenvalues into discrete values (k) and construct a histogram with width k . The training data is then traversed, and the accumulated statistics are counted in the histogram for each discrete value. When selecting a feature, we simply need to traverse the histogram's discrete values to find the optimum segmentation point. GBM removes the level decision tree growth strategy that most GBDT use today, using a leaf-wise approach with limitations on depth. In addition, the level-wise is an inefficient algorithm, because it treats the leaves of the same layer unhindered, bringing a lot of unnecessary overhead.

Multilayer Perceptron

Multilayer perceptron is feed forward artificial neural network that composed of minimum 3 layers of nodes: input layer, hidden layer and output layer. Using activation feature for every layer. Activation method measures the weighted number of the inputs and applies bias to it. It helps one to determine which neuron will be excluded and not included in external relations (Varmedja et al., 2019).

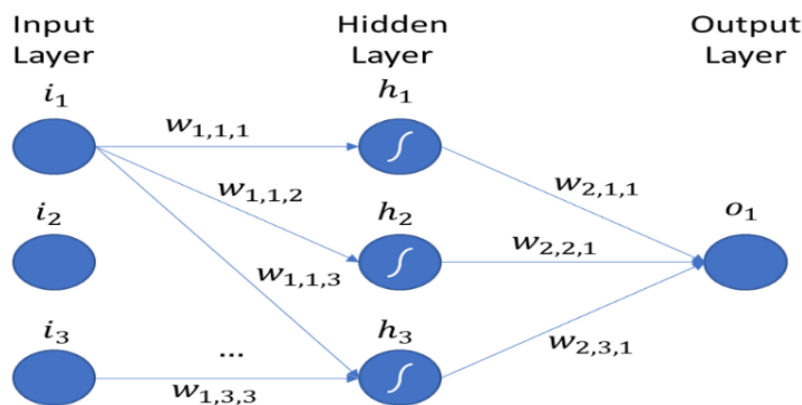


Figure 1: show the multilayer Perceptron architecture

(Source from <https://www.playandlearntocode.com>)

Support Vector Machine

SVM is one of the most common algorithms for regression, classification and machine learning. It is a supervised learning algorithm that analyzes the data used for

classification and regression. SVM modeling requires two steps, first to train a data set and then to acquire a model, and then to use this model to predict the details of the test data set. A Support Vector Machine (SVM) is a discriminative classifier formally described by a separating hyperplane where the SVM model represents the training data points as space points and mapping is performed such that points of different classes are separated by the widest possible distance. For new data points, mapping is performed in the same space and then estimated to which side of the distance they land. Plotting is conducted in the SVM algorithm when each data object is taken as a point in the n dimensional space where n is a number of features, with the value of a particular coordinate being the value of each function. Then, by finding the hyperplane that divides the two groups very well, classification is carried out. (Khare and Yunus 2018).

Fuzzy Rule

It is used in situations where we have no discrete values of fact, i.e. if they are continuous. This is a multi-valued logic. There are some rules on the grounds on which a transaction is classified as a genuine transaction or a fraud. In fuzzy logic there are three important components which must be executed in the order stated:

- Fuzzification
- Rule Based
- Defuzzification

In the case of fuzzification, we identify the incoming transaction according to the monetary value associated with the transaction in the categories of high, low or medium. Rule based covers the drafting of rules based on customer behavior. The transaction is permitted if it meets the given set of rules (Ali Shukur and Kurnaz 2019). In Defuzzification, if the transaction does not comply with the predefined set of rules, it is not allowed to occur. It is automatically halted and then cross-checked with the consumer to see if permission to proceed or be aborted will be given (John and Naaz 2019).

Decision Tree

Decision tree is a type of supervised learning algorithm (which has a predefined target variable) used mostly in classification problems (Lakshmi and Kavila 2018). It is a numerical classification and prediction method. The tree consists of internal nodes denoting the attribute test, each branch denoting the test result, and each leaf node (terminal node) holding the class label. It repetitively partitions a dataset using either the first greedy approach to depth or the first greedy approach to breadth and stops when all the elements are assigned a specific class. To be successful

the partition rule, the data must be divided into groups where a single class predominates in each group. In other words, they are clearly disjointed to a maximum amount (Jain et al., S. 2019).

Types of decision tree

Categorical Variable Decision Tree

Decision Tree which has categorical target variable then it called as categorical variable decision tree.

Continuous Variable Decision Tree

Decision Tree has continuous target variable then it is called as Continuous Variable Decision Tree

Tools

The list of tools used to explore credit card fraud detection analysis is as: This proposed model is implemented in Python. NumPy and Pandas are used for simpler tasks such as data storage and transformation. For data analysis and visualization Matplotlib is used. Seaborn is used for statistical data visualization and for algorithms we used Skit-learn (Kulkarni et al., 2019).

Dataset

Collection of dataset and pre-processing

The dataset is acquired from the Kaggle which hosts the dataset from credit card fraud detections (Tripathi et al., 2018). The dataset is crafted from the MasterCard transactions of European cardholders on Sept 2013. The transactions that occurred for 2 days were recorded that amount to 284,807 entries. The positive category (fraud cases) conjures 0.172% of the transaction's information. The features are transformed and are reduced to 28 principal components as PCA is applied on them and are transformed into numerical input values. These principal components are named as V1, V2, V3 ... and V28. The features include credit limit, gender, marital status, previous months bills, previous months payments, status of existing account, salary assignments, credit history, other credits existing, purpose, credit amount, present employment, savings account, personal status, other debtors, property, age in months, Housing, number of existing credits, Job, Telephone, foreign worker, ID, Credit card number, PIN, Time, Amount and Class.

Table 1: Some important Attributes of European dataset

F. No	Feature	Description
1	Time	Time in seconds to specify the elapses between the current transaction and first transaction
2	Amount	Transaction amount
3	Class	0 – Not Fraud 1 – Fraud

Model Performance Metrics

The basic performance measures derived from the confusion matrix. The confusion matrix is a 2 by 2 matrix table contains four outcomes produced by the binary classifier. Various measures such as sensitivity, specificity, accuracy and error rate are derived from the confusion matrix. fraud is considered as positive class and legal as negative class and hence the meaning of the terms P, N, TP, TN, FP, and FN are defined as follows: positives(P): number of fraud transactions; negatives(N): number of legal transactions (Seeja and Zareapoor 2014).

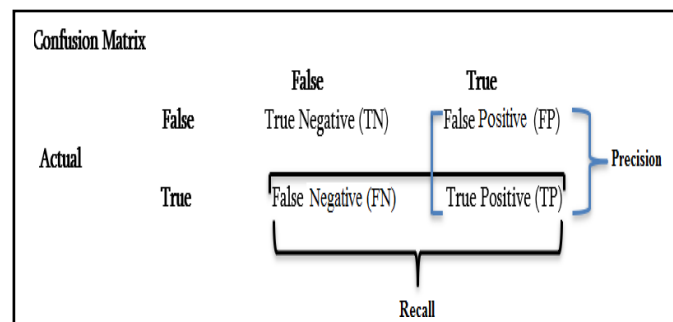


Figure 2: show the confusion matrix component

Table 2: shows confusion matrix of Trained Data and Testing Data

		Meaning
TP	True Positive	number of fraud transactions predicted as fraud
TN	True Negative	number of legal transactions predicted as legal
FP	False Positive	number of legal transactions predicted as fraud
FN	False Negative	number of fraud transactions predicted as legal

Table 3: Confusion Matrix performance measures

No	Measure	Definition	formula
1	Accuracy	Accuracy is also known as the ratio of total number of transactions predicted which are right (Itoo et al., 2020).	$\frac{TP + TN}{TP + FP + TN + FN}$
2	Recall	The proportion of positive values accurately estimated as positive (Itoo et al., 2020).	$\frac{TP}{TP + FN}$
3	Precision	Precision also known as the detection rate is the number of normal or Fraudulent transactions that have been correctly classified (Jain et al., 2019).	$\frac{TP}{TP + FP}$
4	F1- Score	F1 score is harmonic mean of precision and recall. Value of F1 score lies between 0 to 1. Higher F1 score indicates good model (Hordri et al., 2018).	$2 \cdot \frac{Precision \cdot Recall}{Precision + Recall}$

Experimental Results and Discussions

In this study, seven classifier models have been developed and applied for features selection. To evaluate the efficacy of the proposed methods for detecting fraud in credit cards, the proposed method is evaluated using seven algorithms. In order to evaluate these models, training and testing data were used for validation and testing. Recall, precision is used to evaluate the performance of the six classifiers.

We will investigate whether fraudulent transactions occur more frequently within a specific time period.

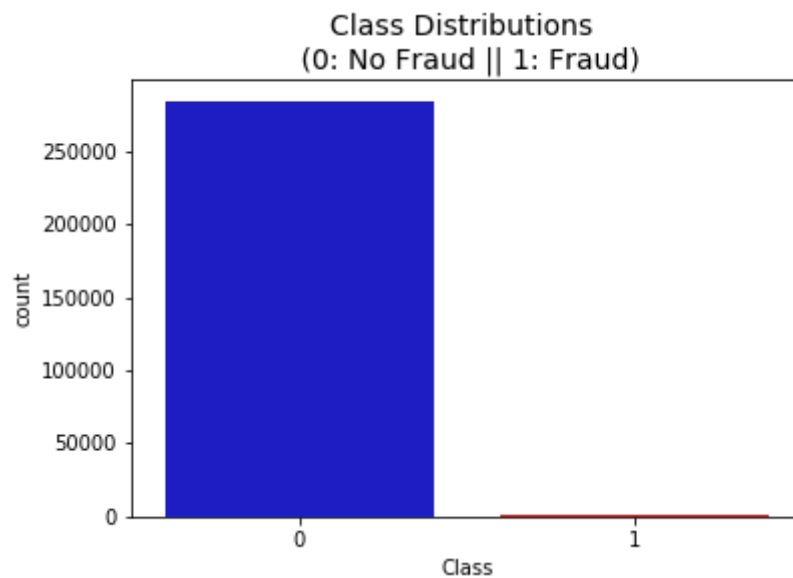


Figure 3: Transaction Class Distribution in Dataset.

This graph shows that the number of fraudulent transactions is much lower than the legitimate ones.

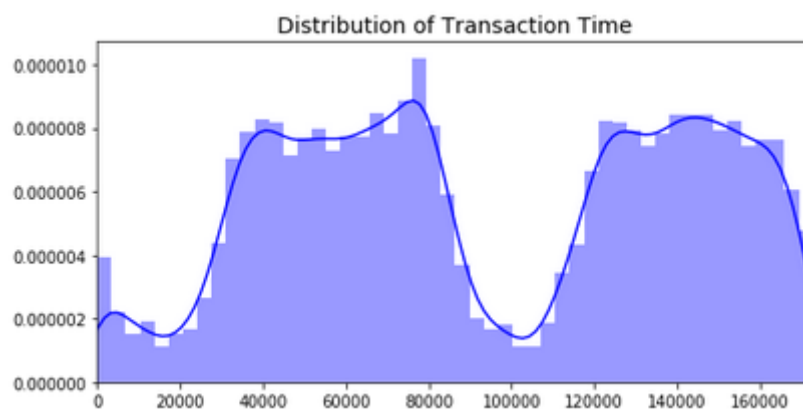


Figure 4: Shows the Distribution of Transaction Time

This graph indicates the times at which transactions were done within two days. It can be shown that the lowest amount of transactions is made during night time and highest during the days.

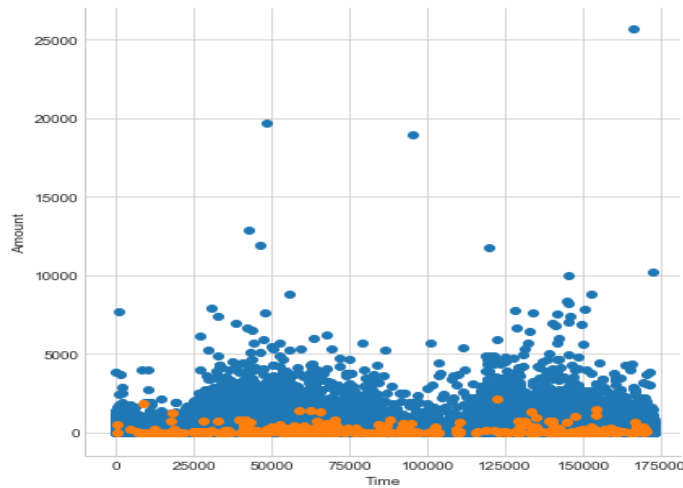


Figure 5: This graph illustrates that the number of illegal transactions is significantly smaller than the number of legal transactions

Table 4: illustrates the results of proposed algorithms using Genetic Algorithms for Features Selection

Algorithm	measurement		
	Recall (False Alarm Rate)	Precision (Detecting Rate)	F-measurement
Random Forest	78.32%	95.73%	86.15%
Gradient Boosted	76.92%	90.16%	83.02%
Logistic Regression	62.24%	91.75%	74.17%
Multilayer Perceptron	74.83%	89.92%	81.68%
Decision Tree	74.83%	95.54%	83.92%
Support Vector Machine	66.67%	88.50%	76.05%
Fuzzy Rule	73.53%	94.34%	82.64%

This table illustrates that Random Forest score highest percentage in Recall (78.32%), Precision (95.73%) and F-measurement (86.15%) compared to proposed algorithms. Gradient Boosted score second percentage in Recall while Decision Tree achieved second percentage followed by Fuzzy Rule in Precision measurement; The Multilayer Perceptron and Decision Tree had the same percentage in Recall while the Logistic Regression had the lowest percentage in Recall measurement. Random Forest and Decision Tree score highest percentage in Precision measurement. In F-measurement Random Forest achieved the highest percentage (86.15%); Gradient Boosted and Decision Tree achieved the 83.02% and 83.92% consequently.

Table 5: illustrates the results of proposed algorithms using Backward Feature Elimination for Features Selection.

Algorithm	measurement		
	Recall (False Alarm Rate)	Precision (Detecting Rate)	F-measurement
Random Forest	76.16%	95.83%	84.87%
Gradient Boosted	77.48%	82.98%	80.14%
Logistic Regression	64.90%	89.09%	75.10%
Multilayer Perceptron	78.15%	86.13%	81.94%
Decision Tree	79.47%	90.23%	84.51%
Support Vector Machine	81.25%	82.98%	82.11%
Fuzzy Rule	68.87%	97.20%	80.62%

Table 5 illustrates that Support Vector Machine achieved highest percentage in Recall (81.25%) compared to proposed algorithms. Decision Tree achieved second percentage in Recall.

The Multilayer Perceptron and Decision Tree had the same percentage in Recall while the Logistic Regression had the lowest percentage in Recall measurement. Fuzzy Rule and Random Forest achieved 97.20%, 95.83% consequently in precision measurement. In F-measurement Random Forest and Decision Tree achieved the highest percentage 84.87%, 84.51% consequently.

Precision, Recall and F-measurement are relevant indicators used when paired with unbalanced data. Precision shows the correctness of the suitability of the test system and the vicinity of the expected result, whereas Recall is a measurement of the total of relevant results. Higher Recall value represents a low false negative (FN) rate, while high Precision indicates a low false positive (FP) rate. High ratings for Precision and Recall show that the classifier restores performance with good accuracy and preserves much of the positive outcomes (Taha and Malebary 2020). As shown in table 4, the Random Forest achieved d highest percentage in Recall, F-measurement and Precision using Genetic Algorithms for Features Selection compared to proposed algorithms. Gradient Boosted achieved second percentage in Recall; The Multilayer Perceptron and Decision Tree had the same percentage in Recall while the Logistic Regression achieved 62.24% which was the lowest percentage in Recall measurement. In F-measurement Random Forest achieved the highest percentage (86.15%); Gradient Boosted and Decision Tree achieved the 83.02% and 83.92% Consequently. Decision Tree achieved 95.54% in Recall followed by Fuzzy Rule 94.34% when using Genetic Algorithms for Features Selection. Table 5 illustrate that Support Vector Machine achieved highest percentage in Recall compared to proposed algorithms when using Backward Feature Elimination for Features Selection. The Multilayer Perceptron and Decision Tree achieved the same percentage 74.83% in Recall while the Logistic Regression had the lowest percentage in Recall measurement. Fuzzy Rule and Random Forest achieved 97.20%, 95.83% consequently in precision measurement. In F-measurement Random Forest and Decision Tree achieved the highest percentage 84.87%, 84.51% consequently. From the results obtained, it is clear that Random Forest obtained average Recall scores 86.15% and 84.87% for Genetic Algorithms and Feature Elimination for Features Selection respectively, among all algorithms, indicating the ability of Random Forest correctly detect more than 86% of the suspicious credit card transactions with a low false-negative percentage in Genetic Algorithms and more than 84% in Feature Elimination for Features Selection. According to the previous results and when comparing the two methods of feature selection, both methods proved effective in selecting the feature, with a clear preference for the genetic algorithm method in the F-measurement with 86.15% for the F-measurement. While Backward Feature Elimination for Features Selection, it had preference in both the Recall and Precision measurement, as the recall rate was 81.25% and 97.20% for Precision. Because it is a combination of the two measures of Recall and Precision, the F-measurement may be regarded the optimum metric for comparing the two approaches and clarifying which of the two methods is better than the other. The method of using genetic algorithms for feature selection had a higher F- measurement than the method of Backward Feature Elimination, with 86.15% for genetic algorithms and 84.87% Backward

Feature Elimination. This leads us to the conclusion that genetic algorithms are the best approach to choose a feature from a dataset and the Random Forest algorithm was the best algorithm compared to proposed algorithms.

Conclusion

In this paper, we have proposed the use of seven classifiers to detect fraudulent credit card transactions. This choice was made by evaluating different methods, including Random Forest, Logistic Regression, Gradient Boosted, Multi Layers Perceptron, *Support Vector Machine*, Decision Tree and Fuzzy Rule, in terms of recall, precision and F- measurement. Compared to the proposed algorithms with two feature selections, according to the previous results and when comparing the two methods of feature selection, both methods proved effective in selecting the feature, with a clear preference for the genetic algorithm method in the F-measurement with 86.15% for the F-measurement. Random Forest and Decision Tree were effective in achieving higher precision; it indicates that there is a more than 95% chance of actually predicting a fraudulent transaction. While Support Vector Machine is more effective at recall among all algorithms, and that implies that there is an 81% probability that the expected fraudulent transaction will actually be true. Compared to proposed algorithms with two feature selections, we suggest that the Random Forest is best algorithm, and more effective in F-measurement with 86% degree. In conclusion, genetic algorithms are the best technique for selecting a feature from a dataset, and the Random Forest algorithm outperformed the other proposed algorithms. In future research, we must develop a new architecture capable of identifying fraudulent transactions in real time by integrating most of modern machine learning techniques.

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On Proof of Sharp Sobolev Embeddings

Isam Eldin Ishag Idris

Department of Mathematics, Faculty of Education, University of Kordofan, Sudan

Corresponding mail: isamishag018@gmail.com

Abstract

This study aimed to present an elementary proof of sharp Sobolev embeddings theorems, it followed the deductive method, and the study found that if we introduce a new function space and use it to improve the limiting Sobolev embedding theorem, we will get a similar results as in Brézis and Wainger studies.

Keywords: Embeddings, Sharp, Sobolev

المستخلص

هدفت هذه الدراسة إلى عرض إثبات أولى لنظرية تضمينات سوبوليف الحدية و اتبعت الدراسة المنهج الإستنباطي و توصلت إلى إنه إذا عرفنا فضاء جديد للدالة وإستخدامناه لتحسين نهاية نظرية تضمين سوبوليف سنحصل على نتائج مماثلة لنتائج دراسات واينجرو بريزيس.

Introduction

Let Ω be an open subset of $\mathbb{N}^{(1-2\epsilon)}$, where $\epsilon \geq -\frac{1}{2}$, let $0 \leq \epsilon < \infty$ and let $W^{(1,1+\epsilon)}(\Omega)$ be the Sobolev space, that is, the set of all functions in $L^{(1+\epsilon)}(\Omega)$, whose distributional derivatives of the first order belong to $L^{(1+\epsilon)}(\Omega)$, too. If $\epsilon = 0$ we assume that $|\Omega| < \infty$. We define $W^{(1,1+\epsilon)}(\Omega)$ as the closure of $C_0^\infty(\Omega)$ in $W^{(1,1+\epsilon)}(\Omega)$. We denote

$(1 + \epsilon)^* = 2\epsilon^2 - \epsilon + 1, 0 \leq \frac{-\epsilon}{2} < \epsilon$. The classical Sobolev theorem (Sobolev, 1963) asserts that

$$W_0^{(1,1+\epsilon)}(\Omega) \hookrightarrow L^{(1+\epsilon)^*}(\Omega) \text{ when } 0 < \frac{-\epsilon}{2} < \epsilon. \quad (1.1)$$

(As usual, \hookrightarrow stands for a continuous embedding.) Although $(1 + \epsilon)^*$ tends to infinity as $(1 + \epsilon) \rightarrow (1 - 2\epsilon) -$, the space $W^{(1,1-2\epsilon)}(\Omega)$ contains unbounded functions. Instead of an embedding into $L^\infty(\Omega)$, (De Năpoli, 2014) one has

$$W_0^{(1,1+\epsilon)}(\Omega) \hookrightarrow \exp L^{-\left(\frac{1-2\epsilon}{2\epsilon}\right)}(\Omega) \quad (1.2)$$

where $\exp L^{-\left(\frac{1-2\epsilon}{2\epsilon}\right)}(\Omega)$ is the exponential-type Orlicz space endowed with the norm

$$\|u\|_{\exp L^{-\left(\frac{1-2\epsilon}{2\epsilon}\right)}(\Omega)} = \inf \left\{ \lambda > 0, \int_{\Omega} \exp \left(\left(\frac{|u|(x)}{\lambda} \right)^{-\left(\frac{1-2\epsilon}{2\epsilon}\right)} \right) dx \leq 1 \right\}.$$

The embedding (1.2) is usually associated to (Trudinger, 1967), similar results had been obtained earlier by (Pokhozhaev, 1965) and (Yudovich, 1961). Now, both of the embeddings (1.1) and (1.2) are sharp within the context of Orlicz spaces. In other words, neither of the target spaces can be replaced by an essentially smaller Orlicz space. This fact was observed by (Cianchi, 1996). However, both of the target spaces can be improved if we are willing to allow different function spaces than Orlicz spaces. Consider Lorentz spaces $L^{(1+\epsilon, 2+\epsilon)}(\Omega)$, defined by

$$L^{(1+\epsilon, 2+\epsilon)}(\Omega) = \left\{ u \text{ measurable on } \Omega, \|u\|_{L^{(1+\epsilon, 2+\epsilon)}(\Omega)} = \int_0^\infty u^*(2-\epsilon) \lambda^{-1+\epsilon} d\lambda < \infty, \text{ where } -1 < \epsilon \leq \infty, -2 < \epsilon \leq \infty \right.$$

and

$$u^*(2-\epsilon) = \inf\{\lambda > 0, |\{x \in \Omega, |u(x)| > \lambda\}| \leq (2-\epsilon)\}, 2 > \epsilon > \infty,$$

is the non-increasing rearrangement of u . Then (1.1) can be replaced by

$$W_0^{(1, 1+\epsilon)}(\Omega) \hookrightarrow L^{((1+\epsilon)^*, (1+\epsilon))}(\Omega) \text{ when } 0 < \frac{-\epsilon}{2} < \epsilon, \quad (1.3)$$

which is a sharper inclusion than (1.1), since $L^{((1+\epsilon)^*, (1+\epsilon))}(\Omega) \subsetneq L^{(1+\epsilon)^*}(\Omega)$. This was observed by (O'Neil, 1963) and (Peetre, 1966). An analogous improvement of (1.2) is possible, too, but Lorentz spaces no longer suffice to do the job; we need a more complicated function space. Namely, we have

$$W_0^{(1, 1+\epsilon)}(\Omega) \hookrightarrow BW_{(1-2\epsilon)}(\Omega), \quad (1.4)$$

$$\text{where } \|u\|_{BW_{(1+\epsilon)}(\Omega)} = \left(\int_0^{|\Omega|} \left(\frac{u^*(2-\epsilon)}{\log\left(\frac{|\Omega|}{2-\epsilon}\right)} \right)^{(1+\epsilon)} \frac{d(2-\epsilon)}{2-\epsilon} \right)^{\frac{1}{1+\epsilon}} \text{ for } -1 < \epsilon <$$

∞

(Malý et al., 2001).

Again, (1.4) is a sharper result than (1.2) as $BW_{(1-2\epsilon)}(\Omega) \subsetneq \exp L^{-\left(\frac{1-2\epsilon}{2\epsilon}\right)}(\Omega)$. The embedding (1.4) is due to (Brézis et al., 1980) and independently to (Hansson, 1979). It can also be derived from capacity estimates of (Maz'ya, 1975). It follows from (Theorem 5.11, Edmunds et

al.,2000) that both (1.3) and (1.4) are sharp within the context of rearrangement-invariant Banach function space. We first establish the weak version of Sobolev-Gagliardo-Nirenberg emedding, that is,

$$\lambda(|\{|u| \geq \lambda\}|)^{\frac{1}{1-2\epsilon}} \leq C \int_{\Omega} |\nabla u| dx \quad (1.5)$$

for all $u \in W_0^{1,1}(\Omega)$ and $\lambda > 0$. It is natural to make profit of this knowledge when deriving (1.5). This approach seems to be less tricky than the traditional way of proving strong-type Gagliardo-Nirenberg estimates. Now, having (1.5) at our disposal, the remaining job, namely the proof of (1.3) and (1.4), can be made very simple and elementary, and it is not essentially more difficult to obtain sharp embeddings than the classical $L^{(2+\epsilon)}$ -ones. For the sake of simplicity we restrict ourselves to the space $W_0^{(1,1+\epsilon)}(\Omega)$. If the domain verifies additional boundedness and regularity assumptions, then the argument can be easily modified to obtain embedding theorems for $W^{(1,1+\epsilon)}(\Omega)$ or Poincaré type inequalities. Now when we know that (1.3) and (1.4) are sharp in the context of rearrangement-inavariant Banach function spaces, we can rewrite them in the form of inequalities. For their formulation and proof, we can even forget the notion of a Lorentz space (Malý et al.,2001).

Theorem A. Assume that $0 \leq \frac{-\epsilon}{2} < \epsilon$. Then

$$\int_0^{|\Omega|} (2 - \epsilon)^{\frac{(1+\epsilon)}{(1+\epsilon)^*} - 1} u^*(2 - \epsilon)^{(1+\epsilon)} d(2 - \epsilon) \leq C \int_{\Omega} |\nabla u(x)|^{(1+\epsilon)} dx \quad (1.6)$$

for all $u \in W_0^{(1,1+\epsilon)}(\Omega)$.

Theorem A. Assume that $|\Omega| < \infty$. Then

$$\int_0^{|\Omega|} \left(\frac{u^*(2-\epsilon)}{\log\left(\frac{|\Omega|}{2-\epsilon}\right)} \right)^{(1-2\epsilon)} \frac{d(2-\epsilon)}{2-\epsilon} \leq C \int_{\Omega} |\nabla u|^{(1-2\epsilon)}(x) dx \quad (1.7)$$

for all $u \in W_0^{(1,1+\epsilon)}(\Omega)$.

Theorem A and B are proved in section 2. Concerning the proofs, we do not claim that our approach is original. A lot of work has been done and our aim is to optimize the argument. In main features we follow the strategy used by (Tartar, 1998), where the inequalities (1.3) and (1.4) (and many others) are also proved. Concerning details, it would be very hard to trace back sources of each particular trick. The fact that strong-type Sobolev estimates can be derived from weak ones has been observed by (Federer, 1996) when deriving the Sobolev-Gagliardo-Nirenberg inequality ($\epsilon = 0$) from the isoperimetric inequality. The co-area integral argument can be simplified by the truncation trick which has been invented by (Maz'ya, 1964) in connection with capacity estimates. Its application to getting strong-type Sobolev embeddings from their weak forms is pursued for example by (Bakry et al., 1995). The point of departure in (Bakry et al., 1995) is the well-known equivalence of the Sobolev embedding to Nash and Moser inequalities, which is avoided here. The goal consists of the introduction and examination of a new function space. In the course of the proof of Theorem B we discover that

when $|\nabla u|^{(1-2\epsilon)}$ is integrable over Ω , then u in fact belongs to a yet smaller class than $BW_{(1-2\epsilon)}(\Omega)$. We denote this class by $W_{(1-2\epsilon)}(\Omega)$. In general, we define $W_{(1+\epsilon)}(\Omega)$ for Ω bounded and $0 \leq \epsilon < \infty$ as the family of all measurable functions on Ω for which

$$\|u\|_{W_{(1+\epsilon)}(\Omega)} := \left(\int_0^{|\Omega|} \frac{\left(u^* \left(\frac{2-\epsilon}{2} \right) - u^*(2-\epsilon) \right)^{(1+\epsilon)}}{2-\epsilon} d(2-\epsilon) \right)^{\frac{1}{1+\epsilon}} < \infty.$$

Note that $u \in W_{(1+\epsilon)}(\Omega)$ if and only if

$$\sum_{\epsilon=0}^{\infty} \left(u^*(2^{-(1-\epsilon)}|\Omega|) - u^*(2^{-\epsilon}|\Omega|) \right)^{(1+\epsilon)} < \infty.$$

In the last we will study the properties of $W_{(1+\epsilon)}(\Omega)$, perhaps the most interesting one is the fact that it is not closed with respect to addition of functions (in particular, it is not a linear set). In view of this, $W_{(1+\epsilon)}(\Omega)$ somewhat resembles the space weak- L^∞ of (Bennett et al., 1981). The space $W_{(1+\epsilon)}(\Omega)$ is interesting from two points of view: first, it is a qualitatively new class of functions which might find its applications in various parts of analysis, and, second, it enables us to obtain a non-trivial improvement of the Sobolev embedding in a limiting case, the sharpest known thus far. At this moment we are not aware of any further connections of this space to practical tasks, but it is one played by weak- L^∞ (cf. Chapter 5,

Section 7, Bennett et al., 1988). Throughout the study, C stands for a positive constant, not necessarily the same at each occurrence.

Elementary proof of embeddings

We proceed in two steps. First we sketch an elementary proof of (1.5) and then show that it implies both (1.6) and (1.7).

Lemma 2.1. For every $u \in W_0^{1,1}(\Omega)$ and $\lambda > 0$, the estimate (1.5) holds.

Proof. For $x \in \mathbb{R}^{(1-2\epsilon)}$ and $R > 0$, denote $B(x, R) = \{y \in \mathbb{R}^{1-2\epsilon}; |y-x| < R\}$, and let $\omega_{1-2\epsilon}$ be the volume of the $1-2\epsilon$ -dimensional unit ball. Then an elementary calculation shows that $\int_{B(0,R)} |x|^{-2\epsilon} dx = \omega_{1-2\epsilon} R^{1-2\epsilon}$. (2.1)

We claim that for every $y \in \mathbb{R}^{(1-2\epsilon)}$ and $G \subset \mathbb{R}^{(1-2\epsilon)}$ of positive finite measure

$$\int_G |x - y|^{-2\epsilon} dx \leq \omega_{(1-2\epsilon)}^{(1-\frac{1}{1-2\epsilon})} |G|^{\frac{1}{1-2\epsilon}}. \quad (2.2)$$

Indeed, fix such G and y and let $B = B(y, R)$ be such that $|B| = |G|$.

Then also $\left|\frac{B}{G}\right| = \left|\frac{G}{B}\right|$, hence

$$\int_{\frac{G}{B}} |x - y|^{-2\epsilon} dx \leq \int_{\frac{G}{B}} R^{-2\epsilon} = \int_{\frac{B}{G}} R^{-2\epsilon} \leq \int_{\frac{B}{G}} |x - y|^{-2\epsilon} dx.$$

Thus, adding $\int_{G \cap B}$ to both sides, we get by (2.1)

$$\begin{aligned} \int_G |x - y|^{-2\epsilon} dx &\leq \int_B |x - y|^{-2\epsilon} dx \leq \omega_{(1-2\epsilon)} R = \omega_{(1-2\epsilon)}^{(1-\frac{1}{1-2\epsilon})} |B|^{\frac{1}{1-2\epsilon}} \\ &= \omega_{(1-2\epsilon)}^{(1-\frac{1}{1-2\epsilon})} |G|^{\frac{1}{1-2\epsilon}}. \end{aligned}$$

Now, fix $u \in C_0^\infty(\Omega)$. Extended by zero outside Ω , we can consider

$u \in C_0^\infty(\mathbb{R}^{(1-2\epsilon)})$. We denote by G the set

$\{x \in \mathbb{R}^{(1-2\epsilon)}; |u(x)| \geq \lambda\}$. Let $K \subset G$ be a compact set. Then, by the well-know integral representation (cf.eg.Chapter 5, Stein, 1970) Fubini's theorem and (2.2),

$$\begin{aligned} \lambda|K| &\leq \int_K |u(x)| dx \\ &\leq C \int_K \int_{\mathbb{R}^{(1-2\epsilon)}} \frac{|\nabla u(y)|}{|x - y|^{-2\epsilon}} dy dx \\ &\leq \int_{\mathbb{R}^{(1-2\epsilon)}} |\nabla u(y)| \int_K \frac{dx}{|x - y|^{-2\epsilon}} dy \\ &\leq C|K|^{\frac{1}{1-2\epsilon}} \int_{\mathbb{R}^{(1-2\epsilon)}} |\nabla u(y)| dy. \end{aligned}$$

Thus, since $|K| < \infty$, $\lambda|K|^{(1-\frac{1}{1-2\epsilon})} \leq C \int_{\mathbb{R}^{(1-2\epsilon)}} |\nabla u(y)| dy$, and in turn, on letting $K \uparrow G$,

$$\lambda|G|^{(1-\frac{1}{1-2\epsilon})} \leq C \int_{\mathbb{R}^{(1-2\epsilon)}} |\nabla u(y)| dy.$$

This shows (1.5) for $u \in C_0^\infty(\Omega)$. The general case follows by a standard approximation argument.

Remark 2.2. The weak estimate (1.5) is usually proved by means of Hardy-Littlewood maximal inequalities based on covering techniques. Our proof avoids the use of covering trikes. Although each step is a rather well-known routine, we were not able to find such a proof in its entirety in the literature. Another elementary proof based on a simple interpolation can be found in (Stein, 1970).

Lemma 2.3. Let $|\Omega| < \infty$ and $0 \leq \frac{-\epsilon}{2} \leq \epsilon$. Let $u \in W_0^{(1,1+\epsilon)}(\Omega)$ and denote

$$(2 - \epsilon)_{(1-\epsilon)} = 2^{-\epsilon} |\Omega| \text{ and } a_{(1-\epsilon)} = u^*((2 - \epsilon)_{(1-\epsilon)}), 1 - \epsilon \in \mathbb{N}. \quad (2.3)$$

Then

$$\sum_{\epsilon=0}^{\infty} (2 - \epsilon)_{(1-\epsilon)}^{\frac{(1+\epsilon)^*}{(1-\epsilon)}} (a_{(-\epsilon+2)} - a_{(1-\epsilon)})^{(1+\epsilon)} \leq C \int_{\Omega} |\nabla u|^{(1+\epsilon)} dx \quad (2.4)$$

with C depending only on $(1 + \epsilon)$ and $(1 - 2\epsilon)$.

Proof. If $\epsilon > 0$, using the Hölder inequality at the right-hand side of (1.5) we obtain

$$\lambda(|\{|u| \geq \lambda\}|)^{\frac{1}{(1-2\epsilon)}} \leq C \left(\int_{\Omega} |\nabla u|^{(1+\epsilon)} dx \right)^{\frac{1}{1+\epsilon}} (|\{|u| > 0\}|)^{\frac{1}{(1+\epsilon)}}, \lambda \in (0, \infty). \quad (2.5)$$

With the convention $\frac{1}{1} = 0$ this also holds for $\epsilon = 0$. Now, given $0 < a < b < \infty$, we use a smooth function φ_a^b on \mathbb{R} such that

$$\begin{cases} \varphi_a^b(s) = 0 & \text{for } -\infty < s \leq a, \\ 0 < (\varphi_a^b)'(s) < 2 & \text{for } a < s < b, \\ \varphi_a^b(s) = b - a & \text{for } b < s \leq \infty. \end{cases}$$

Applying (2.5) to the function $\varphi_a^b \circ |u|$ with $\lambda = (b - a)$, we arrive at

$$(b - a) (|\{|u| \geq b\}|)^{\frac{1}{(1-2\epsilon)'}} \leq C \left(\int_{a < |u| < b} |\nabla u|^{(1+\epsilon)} dx \right)^{\frac{1}{1+\epsilon}} (|\{|u| > 0\}|)^{\frac{1}{(1+\epsilon)'}}. \quad (2.6)$$

Then $|\{|u| > a_{(1-\epsilon)}\}| \leq a_{(1-\epsilon)} \leq |\{|u| \geq a_{(1-\epsilon)}\}|$, and thus (2.6) applied to $a = a_{(1-\epsilon)}$ and $b = a_{(-\epsilon+2)}$ yields

$$(2 - \epsilon)^{\frac{1}{(1-2\epsilon)'}} (a_{(-\epsilon+2)} - a_{(1-\epsilon)}) \leq C \left(\int_{a_{(1-\epsilon)} < u < a_{(-\epsilon+2)}} |\nabla u|^{(1+\epsilon)} dx \right)^{\frac{1}{1+\epsilon}} (2 - \epsilon)^{\frac{1}{(1-\epsilon)'}}$$

That is (recall that $2a_{(-\epsilon+2)} - a_{(1-\epsilon)}$,

$$(2 - \epsilon)^{\frac{1}{(1-\epsilon)'}} (a_{(-\epsilon+2)} - a_{(1-\epsilon)}) \leq C \left(\int_{a_{(1-\epsilon)} < u < a_{(-\epsilon+2)}} |\nabla u|^{(1+\epsilon)} dx \right)^{\frac{1}{1+\epsilon}}$$

where the convention $\left(\frac{1}{(1-2\epsilon)'} = 0\right)$ is used. We raise this estimate to the power $(1 + \epsilon)$ and sum over $(1 - \epsilon)$. We obtain

$$\sum_{\epsilon=0}^{\infty} (2 - \epsilon)^{\frac{(1+\epsilon)}{(1-\epsilon)'}} (a_{(-\epsilon+2)} - a_{(1-\epsilon)})^{(1+\epsilon)} \leq C \sum_{\epsilon=0}^{\infty} \int_{a_{(1-\epsilon)} < u < a_{(-\epsilon+2)}} |\nabla u|^{(1+\epsilon)} dx \leq C \int_{\Omega} |\nabla u|^{(1+\epsilon)} dx, \text{ finishing the proof.}$$

Corollary 2.4. If $|\Omega| < \infty$, then

$$W_0^{(1,1-2\epsilon)}(\Omega) \hookrightarrow W_{(1-2\epsilon)}(\Omega). \quad (2.7),$$

Proof. Note that, for $\epsilon = 0$, (2.4) reads as

$$\sum_{\epsilon=0}^{\infty} (a_{(-\epsilon+2)} - a_{(1-\epsilon)})^{(1-2\epsilon)} \leq C \int_{\Omega} |\nabla u|^{(1-2\epsilon)} dx, \quad (2.8)$$

which is just a discrete version (2.7).

Proof of Theorem A. Let us first assume that $|\Omega| < \infty$ and fix $u \in W_0^{(1,1-2\epsilon)}(\Omega) \cap L^\infty(\Omega)$. Let $(2 - \epsilon)_{(1-\epsilon)}$ and $a_{(1-\epsilon)}$ have the same meaning as in (2.3). Given $\epsilon > 0$, the convexity of $(2 - \epsilon)^{(1+\epsilon)}$ yields (Lemma 1.1, Malý et al., 1997)

$$a_{(-\epsilon+2)}^{(1+\epsilon)} \leq \left(1 + \frac{1}{\epsilon}\right)^\epsilon (a_{(-\epsilon+2)} - a_{(1-\epsilon)})^{(1+\epsilon)} + (1 + \epsilon)^\epsilon a_{(1-\epsilon)}^{(1+\epsilon)}.$$

Hence (taking into account that $a_1 = u^*(2 - \epsilon)_1 = 0$)

$$\begin{aligned} 2^{\frac{(1+\epsilon)}{(1+\epsilon)^*}} \sum_{\epsilon=0}^{\infty} (2 - \epsilon)^{\frac{(1+\epsilon)}{(1+\epsilon)^*}} a_{(-\epsilon+2)}^{(1+\epsilon)} &= \sum_{\epsilon=0}^{\infty} (2 - \epsilon)^{\frac{(1+\epsilon)}{(1+\epsilon)^*}} a_{(-\epsilon+2)}^{(1+\epsilon)} \\ &\leq (1 + \epsilon)^\epsilon \sum_{\epsilon=0}^{\infty} (2 - \epsilon)^{\frac{(1+\epsilon)}{(1+\epsilon)^*}} a_{(1-\epsilon)}^{(1+\epsilon)} \\ &\quad + \left(1 + \frac{1}{\epsilon}\right)^\epsilon \sum_{\epsilon=0}^{\infty} (2 - \epsilon)^{\frac{(1+\epsilon)}{(1+\epsilon)^*}} (a_{(-\epsilon+2)} - a_{(1-\epsilon)})^{(1+\epsilon)}. \end{aligned}$$

Choosing $\epsilon > 0$ so small that $(1 + \epsilon)^\epsilon < 2^{\frac{(1+\epsilon)}{(1+\epsilon)^*}}$ we obtain $\sum_{\epsilon=0}^{\infty} (2 - \epsilon - \epsilon + 2)1 + \epsilon * u * 2 - \epsilon 1 - \epsilon 1 + \epsilon \leq C \Omega \nabla u 1 + \epsilon dx$, which is a discrete version of (1.6). Finally, by a standard truncation and approximation argument, we extend the result to all $u \in W_0^{(1,1+\epsilon)}(\Omega)$ and then to the case when, possibly, $|\Omega| < \infty$ (Pérez, 2006).

Proof of Theorem B. As in the proof of Theorem A, we can restrict ourselves to the case when $u \in W_0^{(1,1+\epsilon)}(\Omega) \cap L^\infty(\Omega)$. We fix such u and assume that $(2 - \epsilon)_{(1-\epsilon)}$ and $a_{(1-\epsilon)}$ have the same meaning as in (2.3). Given $m \in 1 - 2\epsilon$, we have (using $a = 0$)

$$\begin{aligned} 0 \leq \frac{a_m^{(1-2\epsilon)}}{m^{-2\epsilon}} &= \sum_{\epsilon=0}^{m-1} \left(\frac{a_{(-\epsilon+2)}^{(1-2\epsilon)}}{(-\epsilon+2)^{-2\epsilon}} - \frac{a_{(1-\epsilon)}^{(1-2\epsilon)}}{(1-\epsilon)^{-2\epsilon}} \right) \\ &= \sum_{\epsilon=0}^{m-1} \frac{a_{(-\epsilon+2)}^{(1-2\epsilon)} - a_{(1-\epsilon)}^{(1-2\epsilon)}}{(-\epsilon+2)^{n-1}} \\ &\quad - \sum_{\epsilon=0}^{m-1} a_{(1-\epsilon)}^{(1-2\epsilon)} \left(\frac{1}{(1-\epsilon)^{-2\epsilon}} - \frac{1}{(-\epsilon+2)^{-2\epsilon}} \right). \end{aligned}$$

Hence (passing to limit for $m \rightarrow \infty$)

$$\begin{aligned}
 \sum_{\epsilon=0}^{\infty} \frac{a_{(1-\epsilon)}^{(1-2\epsilon)}}{(1-\epsilon)^{(1-2\epsilon)}} &\leq C \sum_{\epsilon=0}^{m-1} a_{(1-\epsilon)}^{(1-2\epsilon)} \left(\frac{1}{(1-\epsilon)^{-2\epsilon}} - \frac{1}{(-\epsilon+2)^{-2\epsilon}} \right) \\
 &\leq C \sum_{\epsilon=0}^{m-1} \frac{a_{(-\epsilon+2)}^{(1-2\epsilon)} - a_{(1-\epsilon)}^{(1-2\epsilon)}}{(-\epsilon+2)^{n-1}} \\
 &\leq C \sum_{\epsilon=0}^{\infty} \frac{a_{(-\epsilon+2)}^{-2\epsilon} (a_{(-\epsilon+2)}^{(1-2\epsilon)} - a_{(1-\epsilon)}^{(1-2\epsilon)})}{(-\epsilon+2)^{-2\epsilon}} \\
 &\leq C \left(\sum_{\epsilon=0}^{\infty} (a_{(-\epsilon+2)} - a_{(1-\epsilon)})^{(1-2\epsilon)} \right)^{\left(\frac{1}{1-2\epsilon}\right)} \left(\sum_{\epsilon=0}^{\infty} \frac{a_{(-\epsilon+2)}^n}{(-\epsilon+2)^{1-2\epsilon}} \right)^{\left(1-\frac{1}{1-2\epsilon}\right)}. \quad (2.9)
 \end{aligned}$$

Recalling that $a_1 = 0$, we infer from (2.8) and (2.9) that

$$\begin{aligned}
 \sum_{\epsilon=0}^{\infty} \frac{u^* ((2-\epsilon)_{1-\epsilon})^{(1-2\epsilon)}}{(1-\epsilon)^{(1-2\epsilon)}} &= \sum_{\epsilon=0}^{\infty} \frac{a_{(1-\epsilon)}^{(1-2\epsilon)}}{(1-\epsilon)^{(1-2\epsilon)}} \\
 &\leq C \sum_{\epsilon}^{\infty} (a_{(-\epsilon+2)} - a_{(1-\epsilon)})^{(1-2\epsilon)} \leq C \int_{\Omega} |\nabla u|^{(1-2\epsilon)} dx,
 \end{aligned}$$

a discrete version of (1.7).

3. The space $W_{(1+\epsilon)}(\Omega)$

Let us now have a closer look at the new function space $W_{(1+\epsilon)}(\Omega)$

Theorem 3.1. Assume that $|\Omega| < \infty$ and $0 \leq \epsilon < \infty$. Then:

- i. $\|\chi_E\|_{W_{(1+\epsilon)}(\Omega)} = (\log 2)^{\frac{1}{1+\epsilon}}$ for every measurable $E \subset \Omega$;
- ii. $L^\infty(\Omega) \not\subseteq W_{(1+\epsilon)}(\Omega)$;



- iii. each integer-valued $u \in W_{(1+\epsilon)}(\Omega)$ is bounded;
- iv. $W_{(1+\epsilon)}(\Omega)$ is not a linear set;
- v. $W_{(1+\epsilon)}(\Omega) \subsetneq BW_{(1+\epsilon)}(\Omega)$

Proof.

- i. for every $E \subset \Omega$, we have $\|\chi_E\|_{W_{(1+\epsilon)}(\Omega)} = \left(\int_{|E|} \frac{d(2-\epsilon)}{2-\epsilon} \right)^{\frac{1}{1+\epsilon}} = \log 211+\epsilon$.
- ii. First, let $\|u\|_{\infty} \leq 1$. Then

$$\begin{aligned}
 & \int_0^{|\Omega|} \frac{\left(u^* \left(\frac{2-\epsilon}{2}\right) - u^*(2-\epsilon)\right)^{(1+\epsilon)}}{2-\epsilon} d(2-\epsilon) \\
 & \leq \int_0^{|\Omega|} \frac{\left(u^* \left(\frac{2-\epsilon}{2}\right) - u^*(2-\epsilon)\right)}{2-\epsilon} d(2-\epsilon) \\
 & = \lim_{\epsilon \rightarrow 0} \int_0^{\frac{1}{\epsilon}} \frac{\left(u^* \left(\frac{2-\epsilon}{2}\right) - u^*(2-\epsilon)\right)}{2-\epsilon} d(2-\epsilon) \\
 & = \lim_{\epsilon \rightarrow 0} \left(\int_0^{\frac{1}{\epsilon}} \frac{u^* \left(\frac{2-\epsilon}{2}\right)}{2-\epsilon} d(2-\epsilon) \right. \\
 & \quad \left. - \int_0^{\frac{1}{\epsilon}} \frac{u^*(2-\epsilon)}{2-\epsilon} d(2-\epsilon) \right) \\
 & = \lim_{\epsilon \rightarrow 0} \left(\int_{\frac{\epsilon}{2}}^{\frac{1}{2\epsilon}} \frac{u^*(2-\epsilon)}{2-\epsilon} d(2-\epsilon) \right. \\
 & \quad \left. - \int_{\epsilon}^{\frac{1}{\epsilon}} \frac{u^*(2-\epsilon)}{2-\epsilon} d(2-\epsilon) \right) \leq \lim_{\epsilon \rightarrow 0} \int_{\frac{\epsilon}{2}}^{\epsilon} \frac{u^*(2-\epsilon)}{2-\epsilon} d(2-\epsilon) \\
 & = (\log 2) \|u\|_{\infty}.
 \end{aligned}$$

Since $W_{(1+\epsilon)}(\Omega)$ is obviously closed with respect to the multiplication by a scalar, we get $L^{\infty}(\Omega) \subset W_{(1+\epsilon)}(\Omega)$. We have to show that this inclusion is strict. To this end, let be any measurable function defined on Ω such that

$$u^*(2-\epsilon) = \left(\log \frac{|\Omega|}{2-\epsilon}\right)^{\alpha}, \quad 2 > \epsilon > -\frac{|\Omega|+4}{2}, \quad \text{with } 0 < \alpha < 1 \text{ to be}$$

determined later. Then

$$\frac{u^*\left(\frac{2-\epsilon}{2}\right)}{2-\epsilon} - u^*(2-\epsilon) = \int_{\frac{1}{2}}^{(2-\epsilon)} \alpha \left(\log \frac{|\Omega|}{1+2\epsilon}\right)^{\alpha-1} \frac{d(1+2\epsilon)}{1+2\epsilon} \leq \alpha \log 2 \left(\log \frac{|\Omega|}{2-\epsilon}\right)^\alpha.$$

Thus, for $0 < \alpha < \frac{\epsilon}{1+\epsilon}$, u is unbounded, but $\|u\|_{W_{(1+\epsilon)}(\Omega)}^{(1+\epsilon)} \leq$

$$C \int_0^{|\Omega|} \left(\log \frac{|\Omega|}{2-\epsilon}\right)^{(1+\epsilon)(\alpha-1)} \frac{d(2-\epsilon)}{2-\epsilon} < \infty.$$

iii. Suppose that u is an integer-valued unbounded function on Ω .

Then there are $\alpha_1 > \alpha_2 \dots > 0$ such that $u^*(\alpha_j -) + u^*(\alpha_j +) \geq$

$1, j \in \mathbb{N}$. For each $j = 1, 2, \dots$, we have $u^*\left(\frac{2-\epsilon}{2}\right) - u^*(2-\epsilon) \geq$

$1, \alpha_j \leq 2-\epsilon \leq 2\alpha_j$, and thus

$$\int_{\alpha_j}^{2\alpha_j} \frac{\left(u^*\left(\frac{2-\epsilon}{2}\right) - u^*(2-\epsilon)\right)^{(1+\epsilon)}}{2-\epsilon} d(2-\epsilon) \geq \int_{\alpha_j}^{2\alpha_j} \frac{d(2-\epsilon)}{2-\epsilon} d(2-\epsilon) \log 2.$$

From the system $\{(\alpha_j, 2\alpha_j)\}$ of intervals we may obviously select an infinite disjoint subsystem, and such

$$\int_0^{|\Omega|} \frac{\left(u^*\left(\frac{2-\epsilon}{2}\right) - u^*(2-\epsilon)\right)^{(1+\epsilon)}}{2-\epsilon} d(2-\epsilon) = \infty, \text{ whence } u \notin W_{(1+\epsilon)}(\Omega)$$

iv. It follows from (ii) that there is a non-negative function $u \in W_{(1+\epsilon)}(\Omega)$ such that u^* is unbounded. Let $[u]$ be the integer part of u and set $\omega := [u] + 1, v := \omega - u$.

Then $|v| \leq 1$, thus by $v \in W_{(1+\epsilon)}(\Omega)$ (ii). On the other hand, $\omega = u + v$ is integer valued but unbounded, hence $u \notin W_{(1+\epsilon)}(\Omega)$ by (iii).



- v. The (discrete version of) the inclusion $W_{(1+\epsilon)}(\Omega) \subset BW_{(1+\epsilon)}(\Omega)$ was shown in the proof of Theorem B. The inclusion is strict indeed, since $BW_{(1+\epsilon)}(\Omega)$ is a linear set, whereas $W_{(1-2\epsilon)}(\Omega)$ is not. Note that we have in fact shown that $W_{(1+\epsilon)}(\Omega) + L^\infty(\Omega)$ (Pick, 2009-2012).

Conclusion

The aim of this study is two-fold. First, we wish to contribute to the discussion on how to teach (sharp) Sobolev embeddings in advanced courses. We have compiled an elementary unified proof, which we are going to show in a self-contained way. We establish the weak version of Sobolev-Gagliardo-Nirenberg embedding. In our opinion, it is not reasonable to avoid potential estimates in lecture courses on function spaces. It is natural to make profit of this knowledge when deriving (1.5). This approach seems to be less tricky than the traditional way of proving strong-type Gagliardo-Nirenberg estimates.

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